

# **Modelica<sup>TM</sup> - A Unified Object-Oriented Language for Physical Systems Modeling**

***LANGUAGE SPECIFICATION***

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# Contents

1	Introduction .....	5
1.1	Overview of Modelica .....	5
1.2	Scope of the specification .....	5
1.3	Definitions and glossary .....	5
2	Modelica syntax .....	7
2.1	Lexical conventions .....	7
2.2	Grammar .....	7
2.2.1	Model definition .....	7
2.2.2	Class definition .....	7
2.2.3	Extends .....	8
2.2.4	Component clause .....	8
2.2.5	Modification .....	8
2.2.6	Equations .....	9
2.2.7	Expressions .....	10
3	Modelica semantics .....	13
3.1	Fundamentals .....	13
3.1.1	Scoping and name lookup .....	13
3.1.1.1	Parents .....	13
3.1.1.2	Static name lookup .....	13
3.1.1.3	Dynamic name lookup .....	14
3.1.2	Environment and modification .....	15
3.1.2.1	Environment .....	15
3.1.2.2	Merging of modifications .....	15
3.1.2.3	Single modification .....	17
3.1.2.4	Instantiation order .....	17
3.1.3	Subtyping and type equivalence .....	17
3.1.3.1	Subtyping of classes .....	17
3.1.3.2	Subtyping of components .....	17
3.1.3.3	Type equivalence .....	17
3.1.3.4	Type identity .....	18
3.1.4	Classes on external files .....	18
3.2	Declarations .....	19
3.2.1	Component clause .....	19
3.2.2	Variability prefix .....	19

3.2.3	Expressions .....	21
3.2.4	Vectors, Matrices, and Arrays.....	22
3.2.4.1	Array declarations .....	22
3.2.4.2	Built-in Functions for Array Expressions.....	24
3.2.4.3	Vector, Matrix and Array Constructors .....	25
	Array Construction .....	25
	Array Concatenation.....	25
	Array Concatenation along First and Second Dimensions.....	26
	Vector Construction.....	27
3.2.4.4	Array access operator .....	27
3.2.4.5	Scalar, vector, matrix, and array operator functions.....	28
	Numeric Type Class.....	28
	Equality and Assignment of type classes .....	28
	Addition and Subtraction of numeric type classes .....	28
	Scalar Multiplication of numeric type classes .....	28
	Matrix Multiplication of numeric type classes.....	29
	Scalar Division of numeric type classes.....	29
	Exponentiation of Scalars of numeric type classes .....	29
	Scalar Exponentiation of Square Matrices of numeric type classes.....	29
	Slice operation .....	29
	Relational operators .....	30
	Functions.....	30
	Empty Arrays.....	31
3.2.5	Final element modification .....	31
3.2.6	Short class definition.....	32
3.2.7	Local class definition .....	32
3.2.8	Extends clause.....	33
3.2.9	Redeclaration .....	34
3.3	Equations .....	35
3.3.1	Equation clause .....	35
3.3.2	If clause .....	35
3.3.3	For clause .....	35
3.3.4	When clause .....	36
3.3.5	Assert .....	36
3.3.6	Connections.....	37
3.3.6.1	Generation of connection equations .....	37
3.3.6.2	Restrictions.....	38

3.4	Functions .....	38
3.5	Code Optimizations .....	39
3.6	Events and Synchronization.....	40
3.7	Restricted classes .....	42
3.8	Variable attributes.....	42
3.9	Intrinsic library functionality .....	43
3.9.1	Built-in variable time .....	43
3.9.2	Modelica built-in operators.....	44
4	Mathematical description of Hybrid DAEs .....	48
5	Unit expressions .....	50
5.1	The Syntax of unit expressions.....	50
5.2	Examples .....	51
6	External function interface .....	52
6.1	Overview .....	52
6.2	Argument type mapping .....	52
6.2.1	Simple types.....	53
6.2.2	Arrays.....	53
6.2.3	Records .....	55
6.3	Return type mapping.....	55
6.4	Aliasing.....	56
6.5	Examples .....	57
6.5.1	Input parameters, function value .....	57
6.5.2	Arbitrary placement of output parameters, no external function value .....	57
6.5.3	External function with both function value and output variable .....	57
7	Modelica standard library.....	59

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# 1 Introduction

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## 1.1 Overview of Modelica

Modelica is a language for modeling of physical systems, designed to support effective library development and model exchange. It is a modern language built on non-causal modeling with mathematical equations and object-oriented constructs to facilitate reuse of modeling knowledge.

## 1.2 Scope of the specification

The Modelica language is specified by means of a set of rules for translating a model described in Modelica to the corresponding model described as a flat hybrid DAE. The key issues of the translation (or instantiation in object-oriented terminology) are:

- Expansion of inherited base classes
- Parameterization of base classes, local classes and components
- Generation of connection equations from **connect** statements

The flat hybrid DAE form consists of:

- Declarations of variables with the appropriate basic types, prefixes and attributes, such as "parameter Real v=5".
- Equations from equation sections.
- Function invocations where an invocation is treated as a set of equations which are functions of all input and of all result variables (number of equations = number of basic result variables).
- Algorithm sections where every section is treated as a set of equations which are functions of the variables occurring in the algorithm section (number of equations = number of different assigned variables).
- When clauses where every when clause is treated as a set of conditionally evaluated equations, also called instantaneous equations, which are functions of the variables occurring in the clause (number of equations = number of different assigned variables).

Therefore, a flat hybrid DAE is seen as a set of equations where some of the equations are only conditionally evaluated (e.g. instantaneous equations are only evaluated when the corresponding when-condition becomes true).

The Modelica specification does not define the result of simulating a model or what constitutes a mathematically well-defined model.

## 1.3 Definitions and glossary

The semantic specification should be read together with the Modelica grammar. Non-normative text, i.e., examples and comments, are enclosed in `[ ]`, comments are set in *italics*.

Term	Definition
Component	An element defined by the production component-clause in the Modelica grammar.

Element	Class definitions, extends-clauses and component-clauses declared in a class.
Instantiation	The translation of a model described in Modelica to the corresponding model described as a hybrid DAE, involving expansion of inherited base classes, parameterization of base classes, local classes and components, and generation of connection equations from <b>connect</b> statements

## 2 Modelica syntax

### 2.1 Lexical conventions

The following syntactic meta symbols are used (extended BNF):

[ ] optional  
 { } repeat zero or more times

The following lexical units are defined:

```

IDENT = NONDIGIT { DIGIT | NONDIGIT }
NONDIGIT = "_" | letters "a" to "z" | letters "A" to "Z"
STRING = "" { S-CHAR | S-ESCAPE } ""
S-CHAR = any member of the source character set except double-quote "", and backslash \
S-ESCAPE = "\'" | "\\"" | "\?" | "\\\\" |
           "\a" | "\b" | "\f" | "\n" | "\r" | "\t" | "\v"
DIGIT = 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
UNSIGNED_INTEGER = DIGIT { DIGIT }
UNSIGNED_NUMBER = UNSIGNED_INTEGER [ "." [ UNSIGNED_INTEGER ] ]
                  [ ( e | E ) [ "+" | "-" ] UNSIGNED_INTEGER ]
  
```

Note: string constant concatenation "a" "b" becoming "ab" (as in C) is replaced by the "+" operator in Modelica.

Modelica uses the same comment syntax as C++ and Java. Inside a comment, the sequence <HTML> ... </HTML> indicates HTML code which may be used by tools to facilitate model documentation.

Keywords and built-in operators of the Modelica language are written in bold face. Keywords are reserved words and may not be used as identifiers.

### 2.2 Grammar

#### 2.2.1 Model definition

```
model_definition:
  { [ final ] class_definition ";" }
```

#### 2.2.2 Class definition

```

class_definition :
  [ partial ]
  ( class | model | record | block | connector | type |
    package | function )
  IDENT class_specifier

class_specifier :
  string_comment composition end IDENT
  | "=" name [ array_subscripts ] [ class_modification ] comment

composition :
  element_list
  { public element_list |
    protected element_list |
    equation_clause |
```

```

    algorithm_clause
}
[ external [ language_specification ]
    [ external_function_call ] ";" [ annotation ";" ] ]

language_specification :
STRING

external_function_call :
[ component_reference "=" ]
IDENT "(" [ expression { "," expression } ] ")"

element_list :
{ element ";" | annotation ";" }

element :
[ inner | outer ]
( [ replaceable ] class_definition | extends_clause | [replaceable]
component_clause )

```

### 2.2.3 Extends

```
extends_clause :
extends name [ class_modification ]
```

### 2.2.4 Component clause

```
component_clause:
type_prefix typeSpecifier [ array_subscripts ] component_list

type_prefix :
[ flow ]
[ discrete | nondiscrete | parameter | constant ] [ input | output ]

typeSpecifier :
name

component_list :
component_declaration { "," component_declaration }

component_declaration :
declaration comment

declaration :
IDENT [ array_subscripts ] [ modification ]
```

### 2.2.5 Modification

```
modification :
class_modification [ "=" expression ]
| "=" expression
| ":=" expression

class_modification :
"(" { argument_list } ")"

argument_list :
argument { "," argument }
```

```

argument :
  element_modification
  | element_redeclaration

element_modification :
  [ final ] component_reference modification

element_redeclaration :
  redeclare
  ([ replaceable ] class_definition | extends_clause | [replaceable]
component_clause1 )

component_clause1 :
  type_prefix type_specifier component_declarator

```

## 2.2.6 Equations

```

equation_clause :
  equation { equation ";" | annotation ";" }

algorithm_clause :
  algorithm { algorithm ";" | annotation ";" }

equation :
  ( simple_expression "=" expression
  | conditional_equation_e
  | for_clause_e
  | when_clause_e
  | connect_clause
  | assert_clause )
  comment

algorithm :
  ( component_reference ( "==" expression | function_call )
  | "(" expression_list ")" "==" function_call
  | conditional_equation_a
  | for_clause_a
  | while_clause
  | when_clause_a
  | assert_clause )
  comment

conditional_equation_e :
  if expression then
  { equation ";" }
  { elseif expression then
  { equation ";" }
  }
  [ else
  { equation ";" }
  ]
  end if

conditional_equation_a :
  if expression then
  { algorithm ";" }

```

```

{ elseif expression then
  { algorithm ";" }
}
[ else
  { algorithm ";" }
]
end if

for_clause_e :
for IDENT in expression loop
  { equation ";" }
end for

for_clause_a :
for IDENT in expression loop
  { algorithm ";" }
end for

while_clause :
while expression loop
  { algorithm ";" }
end while

when_clause_e :
when expression then
  { equation ";" }
end when

when_clause_a :
when expression then
  { algorithm ";" }
end when

connect_clause :
connect "(" connector_ref "," connector_ref ")"

connector_ref :
IDENT [ array_subscripts ] [ ".." IDENT [ array_subscripts ] ]

assert_clause :
assert "(" expression "," STRING { "+" STRING } ")"
terminate "(" STRING { "+" STRING } ")"


```

## 2.2.7 Expressions

```

expression :
  simple_expression
  | if expression then expression else expression

simple_expression :
  logical_expression [ ":" logical_expression [ ":" logical_expression ] ]

logical_expression :
  logical_term { or logical_term }

logical_term :

```

```

logical_factor { and logical_factor }

logical_factor :
  [ not ] relation

relation :
  arithmetic_expression [ rel_op arithmetic_expression ]

rel_op :
  "<" | "<=" | ">" | ">=" | "==" | "<>"

arithmetic_expression :
  [ add_op ] term { add_op term }

add_op :
  "+" | "-"

term :
  factor { mul_op factor }

mul_op :
  "*" | "/"

factor :
  primary [ "^" primary ]

primary :
  UNSIGNED_NUMBER
  | STRING
  | false
  | true
  | component_reference [ function_call ]
  | "(" expression_list ")"
  | "[" expression_list { ";" expression_list } "]"
  | "{" expression_list "}"

name :
  IDENT [ "." name ]

component_reference :
  IDENT [ array_subscripts ] [ "." component_reference ]

function_call :
  "(" function_arguments ")"

function_arguments :
  expression_list
  | named_arguments

named_arguments: [ named_argument { "," named_argument } ]

named_argument: IDENT "=" expression

expression_list :
  expression { "," expression }

array_subscripts :

```

```
"[ " subscript { " , " subscript } " ]"  
subscript :  
  ":" | expression  
  
comment :  
  string_comment [ annotation ]  
  
string_comment :  
  [ STRING { "+" STRING } ]  
  
annotation :  
  annotation class_modification
```

# 3 Modelica semantics

## 3.1 Fundamentals

Instantiation is made in a context which consists of an environment and an ordered set of parents.

### 3.1.1 Scoping and name lookup

#### 3.1.1.1 Parents

The classes lexically enclosing an element form an ordered set of parents. A class defined inside another class definition (the parent) precedes its enclosing class definition in this set.

Enclosing all class definitions is an unnamed parent that contains all top-level class definitions. The order of top-level class definitions in the unnamed parent is undefined.

During instantiation, the parent of an element being instantiated is a partially instantiated class. *[For example, this means that a declaration can refer to a name previously inherited through a previous extends clause.]*

*[Example:*

```
class C1 ... end C1;
class C2 ... end C2;
class C3
  Real x=3;
  C1 y;
  class C4
    Real z;
  end C4;
end C3;
```

*The unnamed parent of class definition C3 contains C1 and C2 in arbitrary order. When instantiating class definition C3, the set of parents of the declaration of x is the partially instantiated class C3 followed by the unnamed parent with C1 and C2. The set of parents of z is C4, C3 and the unnamed parent in that order.]*

#### 3.1.1.2 Static name lookup

Names are looked up at class instantiation to find names of base classes, component types, etc.

For a simple name *[not composed using dot-notation]* lookup is performed as follows:

- When an element, equation or algorithm is instantiated, any name is looked up sequentially in each member of the ordered set of parents until a match is found.

For a composite name of the form A.B *[or A.B.C, etc.]* lookup is performed as follows:

- The first identifier *[A]* is looked up as defined above.
- If the identifier denotes a component, the rest of the name *[e.g., B or B.C]* is looked up in the component.
- If the identifier denotes a class, that class is temporarily instantiated with an empty environment and using the parents of the denoted class. The class must satisfy the requirements for a package. The rest of the name *[e.g., B or B.C]* is looked up in the temporary instantiated class.

*[The temporary class instantiation performed for composite names follow the same rules as class instantiation of the base class in an extends clause, local classes and the type in a component clause, except that the*

*environment is empty.]*

### 3.1.1.3 Dynamic name lookup

An element declared with the prefix **outer** *references* an element instance with the same name but using the prefix **inner** which is nearest in the enclosing *instance* hierarchy of the **outer** element declaration.

There shall exist at least one corresponding **inner** element declaration for an **outer** element reference.*[Inner/outer components may be used to model simple fields, where some physical quantities, such as gravity vector, environment temperature or environment pressure, are accessible from all components in a specific model hierarchy. Inner components are accessible throughout the model, if they are not “shadowed” by a corresponding non-inner declaration in a nested level of the model hierarchy.]*

*[Simple Example:*

```
class A
  outer Real T0;
  ...
end A;

class B
  inner Real T0;
  A a1, a2;      // B.T0, B.a1.T0 and B.a2.T0 is the same variable
  ...
end B;
```

*More complicated example:*

```
class A
  inner Real TI;
  class B
    inner Real TI;
    class C
      Real TI;
      class D
        outer Real TI;  //
      end D;
    end C;
  end B;
end A;

class E
  inner Real TI;
  class F
    inner Real TI;
    class G
      Real TI;
      class H
        A.B.C.D d;
      end H;
    end G;
  end F;
end E;

class I
```

```

inner Real TI;
E           e;    // e.F.G.H.d.TI and e.F.TI is the same variable
E.F.G.H h;    // h.d.TI and TI is the same variable
end I;
]

```

Outer element declarations shall not have modifications. The inner component shall be a subtype of the corresponding outer component. *[If the two types are not identical, the type of the **inner** component defines the instance and the **outer** component references just part of the **inner** component].*

*[Example:*

```

class A
  outer parameter Real p=2;  // error, since modification
end A;

class A
  inner Real TI;
  class B
    outer Integer TI;  // error, since A.TI is no subtype of A.B.TI
  end B;
end A;

```

*Inner declarations can be used to define field functions, such as position dependent gravity fields, e.g.:*

```

function A
  input Real u;
  output Real y;
end A;

function B      // B is a subtype of A
  extends A;
  algorithm
  ...
end B;

class C
  inner function fc = B;    // define function to be actually used
  class D
    outer function fc = A;
    ...
  equation
    y = fc(u);  // function B is used.
  end D;
end C;
]

```

### 3.1.2 Environment and modification

#### 3.1.2.1 Environment

The environment contains arguments which modify elements of the class (e.g., parameter changes). The environment is built by merging class modifications, where outer modifications override inner modifications.

#### 3.1.2.2 Merging of modifications

*[The following larger example demonstrates several aspects:*

```

class C1
  class C11
    parameter Real x;
  end C11;
end C1;
class C2
  class C21
    ...
  end C21;
end C2;
class C3
  extends C1;
  C11 t(x=3);           // ok, C11 has been inherited from C1
  C21 u;                // error, C21 has not yet been inherited
  extends C2;
end C3;

```

The environment of the declaration of *t* is (*x*=3). The environment is built by merging class modifications, as shown by:

```

class C1
  parameter Real a;
end C1;
class C2
  parameter Real b;
end C2;
class C3
  parameter Real x1;           // No default value
  parameter Real x2 = 2;       // Default value 2
  parameter C1 x3;           // No default value for x3.a
  parameter C1 x4(a=4);      // x4.a has default value 4
  extends C1;                // No default value for inherited element a
  extends C2(b=6);          // Inherited b has default value 6
end C3;
class C4
  extends C3(x2=22, x3(a=33), x4(a=44), C1(a=55), b=66);
end C4;

```

Outer modifications override inner modifications, e.g., *b*=66 overrides the nested class modification of **extends** C2(*b*=6). This is known as merging of modifications: **merge((b=66), (b=6))** becomes **(b=66)**.

An instantiation of class C4 will give an object with the following variables:

Variable	Default value
<i>x1</i>	<i>none</i>
<i>x2</i>	22
<i>x3.a</i>	33
<i>x4.a</i>	44
<i>a</i>	55
<i>b</i>	66

The last argument of the C3 modification shows that an inherited element (here, *b*=66) can be directly referred to, without specifying its base class as in C1(*a*=55).]

### 3.1.2.3 Single modification

Two arguments of a modification shall not designate the same primitive attribute of an element. *[Example:*

```

class C1
  Real x[3];
end C1;
class C2 = C1(x=ones(3), x[2]=2); // Error: x[2] designated twice
class C3
  class C4
    Real x;
  end C4;
  C4 a(x(unit = "V"), x = 5.0));
  // Ok, different attributes designated (unit and value)
end C3;

```

]

### 3.1.2.4 Instantiation order

The name of a declared element shall not have the same name as any other element in its partially instantiated parent class. A component shall not have the same name as its type specifier.

The elements of a class are instantiated in the order of declaration. An element is added to its partially instantiated parent class after the complete instantiation of the element. *[For example, Real x = x; is incorrect.]*

## 3.1.3 Subtyping and type equivalence

### 3.1.3.1 Subtyping of classes

For any classes S and C, S is a supertype of C and C is a subtype of S if they are equivalent or if:

- every public declaration element of S also exists in C (according to their names)
- those element types in S are supertypes of the corresponding element types in C.

A base class is the class referred to in an extends clause. The class containing the extends clause is called the derived class. *[Base classes of C are typically supertypes of C, but other classes not related by inheritance can also be supertypes of C.]*

### 3.1.3.2 Subtyping of components

Component B is subtype of A if:

- Both scalars or arrays with the same number of dimensions
- The type of B is subtype of the base type of A (base type for arrays)
- For every dimension of an array
  - The size of A is indefinite, or
  - The value of expression (size of B) - (size of A) is constant equal to 0 (in the environment of B)

### 3.1.3.3 Type equivalence

Two types T and U are equivalent if:

- T and U denote the same built-in type (one of RealType, IntegerType, StringType or BooleanType), or
- T and U are classes, T and U contain the same public declaration elements (according to their names), and the elements types in T are equivalent to the corresponding element types in U.

### 3.1.3.4 Type identity

Two elements T and U are identical if:

- T and U are equivalent,
- they are either both declared as **final** or none is declared **final**,
- for a component their type prefixes are identical, and
- if T and U are classes, T and U contain the same public declaration elements (according to their names), and the elements in T are identical to the corresponding element in U.

### 3.1.3.5 Ordered type identity

Two elements T and U are ordered type identical if and only if:

- T and U are type identical
- If T and U are classes
  - T and U have the same number of elements
  - The i:th declaration element of T and the i:th declaration element of U are ordered type identical

### 3.1.3.6 Function Type Identity

Two functions T and U have identical type if and only if

- T and U have the same number of input and output elements
- For each input or output element
  - The corresponding elements have the same name
  - The corresponding elements are ordered type identical

## 3.1.4 Classes on external files

Class names are automatically mapped to a hierarchical structure of the operating system. Given that A denotes a class at global scope, the name path A.B.C is looked up as follows.

- If A is defined in the current translation unit, the rest of the path (B.C) is looked up inside A.
- Otherwise, A is located in an ordered list of library roots, called MODELICAPATH.

If the name A is a structured entity [*e.g. a directory*], lookup of B.C progresses recursively in A.

If the name A is a non-structured entity [*e.g. a file*], it shall contain only the complete definition of class A, and the rest of the path (B.C) is looked up inside that entity. If the name A is a structured entity [*e.g. a directory with an optional node*], the rest of the path is looked up in the node in the same way as in non-structured entity. If not found, lookup of B.C progresses recursively in A.

*[In a file hierarchy, the node is stored in file package.mo in the package directory].*

*If the node exists it shall contain a class-definition that defines a class with the same name as the structured entity [A]. [The node typically contains documentation and graphical information for a package.]*

Otherwise the lookup fails.

*[On a typical system, MODELICAPATH is an environment variable containing a semicolon-separated list of directory names. Classes are realized by directories with subdirectories, or files containing class definitions. The default file extension for Modelica is .mo; for example, the package A would be stored in file A.mo. If*

*there is both a subdirectory A and a file A.mo, the lookup fails. Other forms of realizing packages are also possible, for example using a hierarchical database.]*

*[The first part of the path A.B.C (i.e., A) is located by searching the ordered list of roots in MODELICAPATH. If no root contains A the lookup fails. If A has been found in one of the roots, the rest of the path is located in A; if that fails, the entire lookup fails without searching for A in any of the remaining roots in MODELICAPATH.]*

## 3.2 Declarations

### 3.2.1 Component clause

If the type specifier of the component denotes a built-in type (RealType, IntegerType, etc.), the instantiated component has the same type.

If the type specifier of the component does not denote a built-in type, the name of the type is looked up (3.1.1). The found type is instantiated with a new environment and the partially instantiated parent of the component. The new environment is the result of merging

- the modification of parent element-modification with the same name as the component
- the modification of the component declaration

in that order.

An environment that defines the value of a component of built-in type is said to define a declaration equation associated with the declared component. For declarations of vectors and matrices, declaration equations are associated with each element. *[This makes it possible to override the declaration equation for a single element in a parent modification, which would not be possible if the declaration equation is regarded as a single matrix equation.]*

Array dimensions shall be non-negative parameter expressions.

Variables declared with the **flow** type prefix shall be a subtype of Real.

Type prefixes (i.e., flow, discrete, nondiscrete, parameter, constant, input, output) shall only be applied for type, record and connector components. Type prefixes of a structured component are also applied to the elements of the component. Type prefixes shall only be applied for a structured component, if no element of the component has a corresponding type prefix of the same category. *[For example, input can only be used, if none of the elements has an input or output type prefix].*

Components of function type may be instantiated. *[A modifier can be used to e.g. change parameters of the function. It is also possible to do such a modification with a class specialization.]* Components of a function do not have start-attributes, but a binding assignment ("`:=`" expression) is an expression such that the component is initialized to this expression at the start of every function invocation (before executing the algorithm section or calling the external function). Binding assignments can only be used for components of a function. If no binding assignment is given for a non-input component its value at the start of the function invocation is undefined. It is a quality of implementation issue to diagnose this for non-external functions. The size of each non-input array component of a function must be given by the inputs. Components of a function will inside the function behave as though they had discrete variability.

### 3.2.2 Variability prefix

The prefixes **nondiscrete**, **discrete**, **parameter**, **constant** of a component declaration are called *variability prefixes* and define in which situation the variable values of a component are initialized (see section 3.6) and when they are changed in *transient* analysis (= solution of initial value problem of the hybrid DAE):

- **Parameter** and **constant** variables vc remain constant during transient analysis (vc=**const.**).
- **Discrete** variables vd are *discrete-time* variables, i.e., they have a vanishing time derivative (**der**(vd)=0) and can change their values only at event instants during transient analysis (see section 3.7).

- **Nondiscrete** variables *vn* are *continuous-time* variables, i.e., they may have a non-vanishing time derivative ( $\text{der}(\text{vn}) \neq 0$  possible) and may change their values at any time during transient analysis (see section 3.7).

If no variability prefix is present in a declaration, the following default variability is used for the variables of the component according to their base types:

base type	default variability	comment
Real	<b>nondiscrete</b>	<i>continuous-time</i> variable
Boolean, Integer, String	<b>discrete</b>	<i>discrete-time</i> variable

[A *discrete* variable is a piecewise constant signal which changes its values only at event instants during simulation. This prefix is needed in order that special algorithms, such as the algorithm of Pantelides for index reduction, can be applied (it must be known that the time derivative of these variables is identical to zero). Furthermore, memory requirements can be reduced in the simulation environment, if it is known that a component can only change at event instants.

A **parameter** variable is constant during simulation. This prefix gives the library designer the possibility to express that the physical equations in a library are only valid if some of the used components are constant during simulation. The same also holds for the *discrete* and *constant* prefix. Additionally, the **parameter** prefix allows a convenient graphical user interface in an experiment environment, to support quick changes of the most important constants of a compiled model. In combination with an *if*-clause, a **parameter** prefix allows to remove parts of a model before the symbolic processing of a model takes place in order to avoid variable causalities in the model (similar to `#ifdef` in C). Class parameters can be sometimes used as an alternative. Example:

```
model Inertia
  parameter Boolean state = true;
  ...
  equation
    J*a = t1 - t2;
    if state then      // code which is removed during symbolic
      der(v) = a;      // processing, if state=false
      der(r) = v;
    end if
  end Inertia;
```

A **constant** variable is similar to a parameter with the difference that constants cannot be changed after they have been declared. It can be used to represent mathematical constants, e.g.

```
constant Real PI=4*arctan(1);
```

A **nondiscrete** Boolean is a continuous-time variable, i.e., its value can change during continuous integration. This type is needed in some rare cases:

```
Boolean off1, off1a;
nondiscrete Boolean off2;
equation
  off1 = s1 < 0;
  off1a = noEvent(s1 < 0);    // error, since off1a is discrete
  off2 = noEvent(s2 < 0);    // possible, because nondiscrete variable
  u1 = if off1 then s1 else 0; // state events
  u2 = if off2 then s2 else 0; // no state events
```

Since *off1* is a *discrete* variable, state events are generated such that *off1* is only changed at event instants. Variable *off2* may change its value during continuous integration. As a result, *u1* is guaranteed to be continuous during continuous integration whereas no such guarantee exists for *u2*.

]

For elements of structured entities with variability prefixes the most restrictive of the variability prefix and the variability of the component wins (using the default variability for the component if there is no variability prefix

on the component).

*[Example:*

```
record A
  constant Real pi=3.14;
  Real y;
  Integer i;
end A;
parameter A a;
  // a.pi is a constant
  // a.y and a.i are parameters
nondiscrete A b;
  // b.pi is a constant
  // b.y is a continuous-time variable
  // b.i is a discrete-time variable

```

]

### 3.2.3 Protected variables

All elements defined under the heading **protected** are regarded as protected. All other elements [*i.e., defined under the heading **public**, without headings or in a separate file*] are public [*i.e. not protected*].

If an extends clause is used under the **protected** heading, all elements of the base class become protected elements of the current class. If an extends clause is a **public** element, all elements of the base class are inherited with their own protection. The eventual headings **protected** and **public** from the base class do not affect the consequent elements of the current class (*i.e. headings **protected** and **public** are not inherited*).

The protected element cannot be accessed via dot notation. They may not be modified or redeclared in class modification.

### 3.2.4 Expressions

**Constant** expressions are:

- Real, Integer, Boolean and String literals.
- Real, Integer, Boolean and String variables declared as **constant**.
- Except for the special built-in operators **initial**, **terminal**, **der**, **edge**, **sample**, **pre** and **analysisType** a function or operator with constant subexpressions as argument (and no parameters defined in the function) is a constant expression.

**Parameter** expressions are:

- Constant expressions.
- Real, Integer, Boolean and String variables declared as **parameter**.
- Except for the special built-in operators **initial**, **terminal**, **der**, **edge**, **sample** and **pre** a function or operator with parameter subexpressions is a parameter expression.
- The function **analysisType()** is parameter expression.

**Discrete** expressions are:

- Parameter expressions.
- Real, Integer, Boolean and String variables declared as **discrete**.

- Function calls where all input arguments of the function are **discrete** expressions.
- Expressions where all the subexpressions are **discrete** expressions.
- Expressions in the body of a **when** clause.
- The result of comparing a nondiscrete Real with a numeric value.
- The functions **pre**, **edge**, and **change** result in discrete expressions.
- Expressions in functions behave as though they were **discrete** expressions.

If the value of a constant or parameter expression is either directly or indirectly used as structural expression (i.e. to compute the size of a component or for if-statements with unequal sizes of the branches) it is a quality-of-implementation issue whether any calls of non-built-in functions are allowed as subexpressions. *[The intention is to erase this restriction for Modelica 2.0.]*

Components declared as **constant** shall have an associated declaration equation with a constant expression. The value of a constant cannot be changed after its declaration.

The declaration equation of a **parameter** component and of the base type attributes *[such as start]* needs to be a parameter expression.

*The declaration equation of a **discrete** component needs to be a discrete expression. [Example:*

```
model Constants
  parameter Real p1 = 1;
  constant Real c1 = p1 + 2;    // error, no constant expression
  parameter Real p2 = p1 + 2;
end Constants;

model Test
  Constants c1(p1=3);    // fine
  Constants c2(p2=7);    // fine, declaration equation can be modified
end Test;
```

]

## 3.2.5 Vectors, Matrices, and Arrays

### 3.2.5.1 Array declarations

The Modelica type system includes scalar number, vector, matrix (number of dimensions, `ndim=2`), and arrays of more than two dimensions. *[There is no distinguishing between a row and column vector.]*

The following table shows the two possible forms of declarations and defines the terminology. C is a placeholder for any class, including the built-in type classes Real, Integer, Boolean and String:

Modelica form 1	Modelica form 2	# dimensions	Designation	Explanation
<code>C x;</code>	<code>C x;</code>	0	Scalar	Scalar
<code>C[n] x;</code>	<code>C x[n];</code>	1	Vector	$n$ - Vector
<code>C[n, m] x;</code>	<code>C x[n, m];</code>	2	Matrix	$n \times m$ Matrix
<code>C[n, m, p, ...] x;</code>	<code>C x[m, n, p, ...];</code>	$k$	Array	Array with $k$ dimensions ( $k \geq 0$ ).

*[The number of dimensions and the dimensions sizes are part of the type, and shall be checked for example at redeclarations. Declaration form 1 displays clearly the type of an array, whereas declaration form 2 is the traditional way of array declarations in languages such as Fortran, C, C++ and is more general in some rare situations, e.g., when a square matrix with unknown sizes is declared which cannot be defined with the first form:*

```
Real A[:, size(A,1)]; // square matrix of unknown size (size(A,1) is the size of the first dimension)
```

```
Real[ :, size(A,1) ] A; // error, because A is used before defined
Real[ : ] v1, v2 // vectors v1 and v2 have unknown sizes. The actual sizes may be different.
```

*It is possible to mix the two declaration forms, but it is not recommended*

```
Real[3,2] x[4,5]; // x has type Real[4,5,3,2];
]
```

Zero-valued dimensions are allowed, so C x[ 0 ] ; declares an empty vector and C x[ 0 , 3 ] ; an empty matrix.

*[Special cases:*

Modelica form 1	Modelica form 2	# dimensions	Designation	Explanation
C[1] x;	C x[1];	1	Vector	1 – Vector, representing a scalar
C[1,1] x;	C x[1, 1];	2	Matrix	1 x 1 – Matrix, representing a scalar
C[n,1] x;	C x[n, 1];	2	Matrix	n x 1 – Matrix, representing a column
C[1,n] x;	C x[1, n];	2	Matrix	1 x n – Matrix, representing a row

]

The type of an array of array is the multidimensional array which is constructed by taking the first dimensions from the component declaration and subsequent dimensions from the maximally expanded component type. A type is maximally expanded, if it is either one of the built-in types (Real, Integer, Boolean, String) or it is not a type class. Before operator overloading is applied, a type class of a variable is maximally expanded.

*[Example:*

```
type Voltage = Real(unit = "V");
type Current = Real(unit = " A ");
connector Pin
    Voltage v; // type class of v = Voltage, type of v = Real
    flow Current i; // type class of i = Current, type of i = Real
end Pin;
type MultiPin = Pin[5];

MultiPin[4] p; // type class of p is MultiPin, type of p is Pin[4,5];

type Point = Real[3];
Point p1[10];
Real p2[10,3];
```

*The components p1 and p2 have identical types.*

```
p2[5] = p1[2]+ p2[4]; // equivalent to p2[5,:]=p1[2,:]+p2[4,:]
Real r[3] = p1[2]; // equivalent to r[3]=p1[2,:]
```

]

*[Automatic assertions at simulation time:*

*Let A be a declared array and i be the declared maximum dimension size of the  $d_i$ -dimension, then an assert statement “assert(  $i \geq 0, \dots$  )” is generated provided this assertion cannot be checked at compile time. It is a quality of implementation issue to generate a good error message if the assertion fails.*

*Let A be a declared array and i be an index accessing an index of the  $d_i$ -dimension. Then for every such index-access an assert statement “assert(  $i \geq 1$  and  $i \leq \text{size}(A,d_i), \dots$  )” is generated, provided this assertion cannot be checked at compile time.*

*For efficiency reasons, these implicit assert statement may be optionally suppressed.]*

### 3.2.5.2 Built-in Functions for Array Expressions

The following function *cannot* be used in Modelica, but is utilized below to define other operators

<b>promote(A,n)</b>	Fills dimensions of size 1 from the right to array A upto dimension n, where "n >= ndims(A)" is required. Let C = promote(A,n), with nA=ndims(A), then ndims(C) = n, size(C,j) = size(A,j) for $1 \leq j \leq nA$ , size(C,j) = 1 for $nA+1 \leq j \leq n$ , $C[i_1, \dots, i_n, 1, \dots, 1] = A[i_1, \dots, i_n, 1, \dots, 1]$
---------------------	--

*[Function promote could not be used in Modelica, because the number of dimensions of the return array cannot be determined at compile time if n is a variable. Below, promote is only used for constant n].*

The following built-in functions for array expressions are provided:

Modelica	Explanation
<b>ndims(A)</b>	Returns the number of dimensions k of array expression A, with $k \geq 0$ .
<b>size(A,i)</b>	Returns the size of dimension i of array expression A where i shall be $> 0$ and $\leq \text{ndims}(A)$ .
<b>size(A)</b>	Returns a vector of length $\text{ndims}(A)$ containing the dimension sizes of A.
<b>scalar(A)</b>	Returns the single element of array A. $\text{size}(A,i) = 1$ is required for $1 \leq i \leq \text{ndims}(A)$ .
<b>vector(A)</b>	Returns a 1-vector, if A is a scalar and otherwise returns a vector containing all the elements of the array, provided there is at most one dimension size $> 1$ .
<b>matrix(A)</b>	Returns $\text{promote}(A,2)$ , if A is a scalar or vector and otherwise returns the elements of the first two dimensions as a matrix. $\text{size}(A,i) = 1$ is required for $2 < i \leq \text{ndims}(A)$ .
<b>transpose(A)</b>	Permutes the first two dimensions of array A. It is an error, if array A does not have at least 2 dimensions.
<b>outerProduct(v1,v2)</b>	Returns the outer product of vectors v1 and v2 (= $\text{matrix}(v) * \text{transpose}(\text{matrix}(v))$ ).
<b>identity(n)</b>	Returns the $n \times n$ Integer identity matrix, with ones on the diagonal and zeros at the other places.
<b>diagonal(v)</b>	Returns a square matrix with the elements of vector v on the diagonal and all other elements zero.
<b>zeros(n<sub>1</sub>,n<sub>2</sub>,n<sub>3</sub>,...)</b>	Returns the $n_1 \times n_2 \times n_3 \times \dots$ Integer array with all elements equal to zero ( $n_i \geq 0$ ).
<b>ones(n<sub>1</sub>,n<sub>2</sub>,n<sub>3</sub>,...)</b>	Return the $n_1 \times n_2 \times n_3 \times \dots$ Integer array with all elements equal to one ( $n_i \geq 0$ ).
<b>fill(s,n<sub>1</sub>,n<sub>2</sub>,n<sub>3</sub>, ...)</b>	Returns the $n_1 \times n_2 \times n_3 \times \dots$ array with all elements equal to scalar expression s which has to be a subtype of Real, Integer, Boolean or String ( $n_i \geq 0$ ). The returned array has the same type as s.
<b>linspace(x1,x2,n)</b>	Returns a Real vector with n equally spaced elements, such that $v = \text{linspace}(x1, x2, n)$ , $v[i] = x1 + (x2 - x1) * (i - 1) / (n - 1)$ for $1 \leq i \leq n$ . It is required that $n \geq 2$ .
<b>min(A)</b>	Returns the smallest element of array expression A.
<b>max(A)</b>	Returns the largest element of array expression A.
<b>sum(A)</b>	Returns the sum of all the elements of array expression A.
<b>product(A)</b>	Returns the product of all the elements of array expression A.
<b>symmetric(A)</b>	Returns a matrix where the diagonal elements and the elements above the diagonal are identical to the corresponding elements of matrix A and where the elements below the diagonal are set equal to the elements above the diagonal of A, i.e., $B := \text{symmetric}(A) \rightarrow B[i,j] := A[i,j]$ , if $i \leq j$ , $B[i,j] := A[j,i]$ , if $i > j$ .
<b>cross(x,y)</b>	Returns the cross product of the 3-vectors x and y, i.e. $\text{cross}(x,y) = \text{vector}([x[2]*y[3]-x[3]*y[2]; x[3]*y[1]-x[1]*y[3]; x[1]*y[2]-x[2]*y[1]])$ ;

<b>skew(x)</b>	Returns the $3 \times 3$ skew symmetric matrix associated with a 3-vector, i.e., $\text{cross}(x,y) = \text{skew}(x)*y$ ; $\text{skew}(x) = [0, -x[3], x[2]; x[3], 0, -x[1]; -x[2], x[1], 0];$
----------------	---

[Example:

```
Real x[4,1,6];
size(x,1) = 4;
size(x);           // vector with elements 4, 1, 6
size(2*x+x) = size(x);

Real[3] v1 = fill(1.0, 3);
Real[3,1] m = matrix(v1);
Real[3] v2 = vector(m);

Boolean check[3,4] = fill(true, 3, 4);

]
```

### 3.2.5.3 Vector, Matrix and Array Constructors

#### Array Construction

The constructor function **array(A,B,C,...)** constructs an array from its arguments according to the following rules:

- Size matching: All arguments must have the same sizes, i.e.,  $\text{size}(A) = \text{size}(B) = \text{size}(C) = \dots$
- All arguments must be *type equivalent*. The datatype of the result array is the maximally expanded type of the arguments. The maximally expanded types should be equivalent. Real and Integer subtypes can be mixed resulting in a Real result array where the Integer numbers have been transformed to Real numbers.
- Each application of this constructor function adds a one-sized dimension to the left in the result compared to the dimensions of the argument arrays, i.e.,  $\text{ndims}(\text{array}(A,B,C)) = \text{ndims}(A) + 1 = \text{ndims}(B) + 1, \dots$
- $\{A, B, C, \dots\}$  is a shorthand notation for  $\text{array}(A, B, C, \dots)$ .
- There must be at least one argument [*i.e.*,  $\text{array}()$  or  $\{\}$  is not defined].

[Examples:

```
{1,2,3} is a 3 vector of type Integer.
{ {11,12,13}, {21,22,23} } is a 2x3 matrix of type Integer
{{{1.0, 2.0, 3.0}}} is a 1x1x3 array of type Real.
```

```
Real[3] v = array(1, 2, 3.0);
type Angle = Real(unit="rad");
parameter Angle alpha = 2.0; // type of alpha is Real.
array(alpha, 2, 3.0) is a 3 vector of type Real.
Angle[3] a = {1.0, alpha, 4}; // type of a is Real[3];
]
```

#### Array Concatenation

The function **cat(k,A,B,C,...)** concatenates arrays A,B,C,... along dimension k according to the following rules:

- Arrays A, B, C, ... must have the same number of dimensions, i.e.,  $\text{ndims}(A) = \text{ndims}(B) = \dots$
- Arrays A, B, C, ... must be *type equivalent*. The datatype of the result array is the maximally expanded type of the arguments. The maximally expanded types should be equivalent. Real and Integer subtypes can be mixed resulting in a Real result array where the Integer numbers have been transformed to Real numbers.
- k has to characterize an existing dimension, i.e.,  $1 \leq k \leq \text{ndims}(A) = \text{ndims}(B) = \text{ndims}(C)$ ; k shall be an integer number.

- Size matching: Arrays A, B, C, ... must have identical array sizes with the exception of the size of dimension k, i.e.,  $\text{size}(A,j) = \text{size}(B,j)$ , for  $1 \leq j \leq \text{ndims}(A)$  and  $j \neq k$ .

[Examples:

```
Real[2,3]  r1 = cat(1, {{1.0, 2.0, 3}}, {{4, 5, 6}});
Real[2,6]  r2 = cat(2, r1, 2*r1);
```

]

Concatenation is formally defined according to:

Let  $R = \text{cat}(k, A, B, C, \dots)$ , and let  $n = \text{ndims}(A) = \text{ndims}(B) = \text{ndims}(C) = \dots$ , then

$\text{size}(R,k) = \text{size}(A,k) + \text{size}(B,k) + \text{size}(C,k) + \dots$

$\text{size}(R,j) = \text{size}(A,j) = \text{size}(B,j) = \text{size}(C,j) = \dots$ , for  $1 \leq j \leq n$  and  $j \neq k$ .

$R[i_1, \dots, i_k, \dots, i_n] = A[i_1, \dots, i_k, \dots, i_n]$ , for  $i_k \leq \text{size}(A,k)$ ,

$R[i_1, \dots, i_k, \dots, i_n] = B[i_1, \dots, i_k - \text{size}(A,i), \dots, i_n]$ , for  $i_k \leq \text{size}(A,k) + \text{size}(B,k)$ ,

...

where  $1 \leq i_j \leq \text{size}(R,j)$  for  $1 \leq j \leq n$ .

### Array Concatenation along First and Second Dimensions

For convenience, a special syntax is supported for the concatenation along the first and second dimensions.

- *Concatenation along first dimension:*  
 $[A; B; C; \dots] = \text{cat}(1, \text{promote}(A,n), \text{promote}(B,n), \text{promote}(C,n), \dots)$  where  
 $n = \max(2, \text{ndims}(A), \text{ndims}(B), \text{ndims}(C), \dots)$ . If necessary, 1-sized dimensions are added to the right of A, B, C before the operation is carried out, in order that the operands have the same number of dimensions which will be at least two.
- *Concatenation along second dimension:*  
 $[A, B, C, \dots] = \text{cat}(2, \text{promote}(A,n), \text{promote}(B,n), \text{promote}(C,n), \dots)$  where  
 $n = \max(2, \text{ndims}(A), \text{ndims}(B), \text{ndims}(C), \dots)$ . If necessary, 1-sized dimensions are added to the right of A, B, C before the operation is carried out, especially that each operand has at least two dimensions.
- The two forms can be mixed.  $[..., \dots]$  has higher precedence than  $[\dots; \dots]$ , e.g.,  $[a, b; c, d]$  is parsed as  $[[a,b]; [c,d]]$ .
- $[A] = \text{promote}(A, \max(2, \text{ndims}(A)))$ , i.e.,  $[A] = A$ , if A has 2 or more dimensions, and it is a matrix with the elements of A, if A is a scalar or a vector.
- There must be at least one argument (i.e. [] is not defined)

[Examples:

```
Real s1, s2, v1[n1], v2[n2], M1[m1,n], M2[m2,n], M3[n,m1], M4[n,m2], K1[m1,n,k], K2[m2,n,k];
```

$[v1;v2]$  is a  $(n1+n2) \times 1$  matrix

$[M1;M2]$  is a  $(m1+m2) \times n$  matrix

$[M3,M4]$  is a  $n \times (m1+m2)$  matrix

$[K1;K2]$  is a  $(m1+m2) \times n \times k$  array

$[s1;s2]$  is a  $2 \times 1$  matrix

$[s1,s1]$  is a  $1 \times 2$  matrix

$[s1]$  is a  $1 \times 1$  matrix

$[v1]$  is a  $n1 \times 1$  matrix

```
Real[3] v1 = array(1, 2, 3);
```

```
Real[3] v2 = {4, 5, 6};
```

```
Real[3,2] m1 = [v1, v2];
```

```
Real[3,2] m2 = [v1, [4;5;6]]; // m1 = m2
```

```
Real[2,3] m3 = [1, 2, 3; 4, 5, 6];
```

```
Real[1,3] m4 = [1, 2, 3];
```

```
Real[3,1] m5 =[1; 2; 3];
]
```

## Vector Construction

Vectors can be constructed with the general array constructor, e.g., `Real[3] v = {1, 2, 3}.`

The colon operator of *simple-expression* can be used instead of or in combination with this general constructor to construct Real and Integer vectors. Semantics of the colon operator:

- `j : k` is the Integer vector  $\{j, j+1, \dots, k\}$ , if `j` and `k` are of type Integer.
- `j : k` is the Real vector  $\{j, j+1.0, \dots, n\}$ , with  $n = \text{floor}(k-j)$ , if `j` and/or `k` are of type Real.
- `j : k` is a Real or Integer vector with zero elements, if  $j > k$ .
- `j : d : k` is the Integer vector  $\{j, j+d, \dots, j+n*d\}$ , with  $n = (k-j)/d$ , if `j`, `d`, and `k` are of type Integer.
- `j : d : k` is the Real vector  $\{j, j+d, \dots, j+n*d\}$ , with  $n = \text{floor}((k-j)/d)$ , if `j`, `d`, or `k` are of type Real.
- `j : d : k` is a Real or Integer vector with zero elements, if  $d > 0$  and  $j > k$  or if  $d < 0$  and  $j < k$ .

[Examples:

```
Real v1[5] = 2.7 : 6.8;
Real v2[5] = {2.7, 3.7, 4.7, 5.7, 6.7} // = same as v1
]
```

### 3.2.5.4 Array access operator

Elements of vector, matrix or array variables are accessed with `[ ]`. A colon is used to denote all indices of one dimension. A vector expression can be used to pick out selected rows, columns and elements of vectors, matrices, and arrays. The number of dimensions of the expression is reduced by the number of scalar index arguments.

[Examples:

- `a[:, j]` is a vector of the  $j$ -th column of `a`,
- `a[j : k]` is  $\{a[j], a[j+1], \dots, a[k]\}$ ,
- `a[:,j : k]` is  $[a[:,j], a[:,j+1], \dots, a[:,k]]$ ,
- `v[2:2:8] = v[ {2,4,6,8} ]` .
- if `x` is a vector, `x[1]` is a scalar, but the slice `x[1 : 5]` is a vector (a vector-valued or colon index expression causes a vector to be returned).]

[Examples given the declaration `x[n, m], v[p], z[i, j, p]`:

Expression	# dimensions	Type of value
<code>x[1, 1]</code>	0	Scalar
<code>x[:, 1]</code>	1	$n - \text{Vector}$
<code>x[1, :]</code>	1	$m - \text{Vector}$
<code>v[1:p]</code>	1	$p - \text{Vector}$
<code>x[1:p, :]</code>	2	$p \times m - \text{Matrix}$
<code>x[1:1, :]</code>	2	$1 \times m - \text{"row" matrix}$
<code>x[{1, 3, 5}, :]</code>	2	$3 \times m - \text{Matrix}$

x[:, v]	2	n x k – Matrix
z[:, 3, :]	2	i x p – Matrix
x[scalar([1]), :]	1	m – Vector
x[vector([1]), :]	2	1 x m - "row" matrix

1

### 3.2.5.5 Scalar, vector, matrix, and array operator functions

The mathematical operations defined on scalars, vectors, and matrices are the subject of linear algebra.

In all contexts that require an expression which is a subtype of Real, an expression which is a subtype of Integer can also be used; the Integer expression is automatically converted to Real.

#### Numeric Type Class

The term *numeric class* is used below for a subtype of the Real or Integer type class.

#### Equality and Assignment of type classes

Equality “a=b” and assignment “a:=b” of scalars, vectors, matrices, and arrays is defined element-wise and require both objects to have the same number of dimensions and corresponding dimension sizes. The operands need to be type equivalent.

Type of a	Type of b	Result of a = b	Operation (j=1:n, k=1:m)
Scalar	Scalar	Scalar	a = b
Vector[n]	Vector[n]	Vector[n]	a[j] = b[j]
Matrix[n, m]	Matrix[n, m]	Matrix[n, m]	a[j, k] = b[j, k]
Array[n, m, ...]	Array[n, m, ...]	Array[n, m, ...]	a[j, k, ...] = b[j, k, ...]

#### Addition and Subtraction of numeric type classes

Addition “a+b” and subtraction “a-b” of numeric scalars, vectors, matrices, and arrays is defined element-wise and require size(a) = size(b) and a numeric type class for a and b.

Type of a	Type of b	Result of a +/- b	Operation c := a +/- b (j=1:n, k=1:m)
Scalar	Scalar	Scalar	c := a +/- b
Vector[n]	Vector[n]	Vector[n]	c[j] := a[j] +/- b[j]
Matrix[n, m]	Matrix[n, m]	Matrix[n, m]	c[j, k] := a[j, k] +/- b[j, k]
Array[n, m, ...]	Array[n, m, ...]	Array[n, m, ...]	c [j, k, ...] := a[j, k, ...] +/- b[j, k, ...]

#### Scalar Multiplication of numeric type classes

Scalar multiplication “s\*a” or “a\*s” with numeric scalar s and numeric scalar, vector, matrix or array a is defined element-wise:

Type of s	Type of a	Type of s*a and a*s	Operation c := s*a or c := a*s (j=1:n, k=1:m)
Scalar	Scalar	Scalar	c := s * a
Scalar	Vector [n]	Vector [n]	c[j] := s* a[j]
Scalar	Matrix [n, m]	Matrix [n, m]	c[j, k] := s* a[j, k]
Scalar	Array[n, m, ...]	Array [n, m, ...]	c [j, k, ...] := s*a[j, k, ...]

## Matrix Multiplication of numeric type classes

Multiplication “a\*b” of numeric vectors and matrices is defined only for the following combinations:

Type of a	Type of b	Type of a*b	Operation c := a*b
Vector [n]	Vector [n]	Scalar	$c := \sum_k (a[k]*b[k]), k=1:n$
Vector [n]	Matrix [n, m]	Vector [m]	$c[j] := \sum_k (a[k]*b[k, j]), j=1:m, k=1:n$
Matrix [n, m]	Vector [m]	Vector [n]	$c[j] := \sum_k (a[j, k]*b[k])$
Matrix [n, m]	Matrix [m, p]	Matrix [n, p]	$c[i, j] = \sum_k (a[i, k]*b[k, j]), i=1:n, k=1:m, k=1:p$

[ Example:

```

Real A[3,3], x[3], b[3];
A*x = b;
x*A = b;                                // same as transpose([x])*A*b
[v]*transpose([v])                      // outer product
v*M*v                                    // scalar
transpose([v])*M*v                       // vector with one element
]

```

## Scalar Division of numeric type classes

Division “a/s” of numeric scalars, vectors, matrices, or arrays a and numeric scalars s is defined element-wise.

Type of a	Type of s	Result of a / s	Operation c := a / s (j=1:n, k=1:m)
Scalar	Scalar	Scalar	$c := a / s$
Vector[n]	Scalar	Vector[n]	$c[k] := a[k] / s$
Matrix[n, m]	Scalar	Matrix[n, m]	$c[j, k] := a[j, k] / s$
Array[n, m, ...]	Scalar	Array[n, m, ...]	$c[j, k, ...] := a[j, k, ...] / s$

## Exponentiation of Scalars of numeric type classes

Exponentiation “a^b” is defined as `pow( )` in the C language if both “a” and “b” are scalars of a numeric type class.

## Scalar Exponentiation of Square Matrices of numeric type classes

Exponentiation “a^s” is defined if “a” is a square numeric matrix and “s” is a scalar as a subtype of Integer with  $s \geq 0$ . The exponentiation is done by repeated multiplication (e.g.  $a^3 = a*a*a$ ;  $a^0 = \text{identity}(\text{size}(a,1))$ ;  $a^1 = a$ ).

[Non-Integer exponents are forbidden, because this would require to compute the eigenvalues and eigenvectors of “a” and this is no longer an elementary operation].

## Slice operation

If  $a$  is an array of records and  $m$  is a component of that record, the expression  $a.m$  is interpreted as *slice operation*. It returns the array of components  $\{a[1].m, \dots\}$ .

If  $m$  is also an array component, the slice operation is valid only if  $\text{size}(a[1].m) = \text{size}(a[2].m) = \dots$

## Relational operators

Relational operators  $<$ ,  $\leq$ ,  $>$ ,  $\geq$ ,  $==$ ,  $\neq$ , are only defined for *scalar* arguments. The result is Boolean and is *true* or *false* if the relation is fulfilled or not, respectively.

In relations of the form  $v1 == v2$  or  $v1 \neq v2$ ,  $v1$  or  $v2$  shall not be a subtype of Real. *[The reason for this rule is that relations with Real arguments are transformed to state events (see section Events below) and this transformation becomes unnecessarily complicated for the == and \neq relational operators (e.g. two crossing functions instead of one crossing function needed, epsilon strategy needed even at event instants). Furthermore, testing on equality of Real variables is questionable on machines where the number length in registers is different to number length in main memory].*

Relations of the form “ $v1 \text{ rel\_op } v2$ ”, with  $v1$  and  $v2$  variables and  $\text{rel\_op}$  a relational operator are called elementary relations. If either  $v1$  or  $v2$  or both variables are a subtype of Real, the relation is called a Real elementary relation.

## Functions

Functions with one scalar return value can be applied to arrays element-wise, e.g. if  $A$  is a vector of reals, then  $\sin(A)$  is a vector where each element is the result of applying the function  $\sin$  to the corresponding element in  $A$ .

Consider the expression  $f(\text{arg1}, \dots, \text{argn})$ , an application of the function  $f$  to the arguments  $\text{arg1}, \dots, \text{argn}$  is defined.

For each passed argument, the type of the argument is checked against the type of the corresponding formal parameter of the function.

1. If the types match, nothing is done.
2. If the types do not match, and a type conversion can be applied, it is applied. Continued with step 1.
3. If the types do not match, and no type conversion is applicable, the passed argument type is checked to see if it is an  $n$ -dimensional array of the formal parameter type. If it is not, the function call is invalid. If it is, we call this a foreach argument.
4. For all foreach arguments, the number and sizes of dimensions must match. If they do not match, the function call is invalid. If no foreach argument exists, the function is applied in the normal fashion, and the result has the type specified by the function definition.
5. The result of the function call expression is an  $n$ -dimensional array with the same dimension sizes as the foreach arguments. Each element  $e_{i,\dots,j}$  is the result of applying  $f$  to arguments constructed from the original arguments in the following way.
  - If the argument is not a foreach argument, it is used as-is.
  - If the argument is a foreach argument, the element at index  $[i,\dots,j]$  is used.

If more than one argument is an array, all of them have to be the same size, and they are traversed in parallel.

*[Examples:*

```
sin({a, b, c})      = {sin(a), sin(b), sin(c)}    // argument is a vector
sin([a,b,c])        = [sin(a),sin(b),sin(c)]      // argument may be a matrix
atan({a,b,c},{d,e,f}) = {atan(a,d), atan(b,e), atan(c,f)}
```

*This works even if the function is declared to take an array as one of its arguments. If  $pval$  is defined as a function that takes one argument that is a vector of Reals and returns a Real, then it can be used with an actual argument which is a two-dimensional array (a vector of vectors). The result type in this case will be a vector of Real.*

```
pval([1,2;3,4]) = [pval([1,2]); pval([3,4])]
sin([1,2;3,4])  = [sin({1,2}); sin({3,4})]
```

```

= [sin(1), sin(2); sin(3), sin(4)]
```

```

function Add
  input Real e1, e2;
  output Real sum1;
algorithm
  sum1 := e1 + e2;
end Add;
```

*Add(1, [1, 2, 3]) adds one to each of the elements of the second argument giving the result [2, 3, 4]. However, it is illegal to write 1 + [1, 2, 3], because the rules for the built-in operators are more restrictive.]*

### Empty Arrays

Arrays may have dimension sizes of 0. E.g.

```

Real x[0];           // an empty vector
Real A[0, 3], B[5, 0], C[0, 0]; // empty matrices
```

- Empty matrices can be constructed with the fill function. E.g.  
`Real A[:, :] = fill(0.0, 0, 1) // a Real 0 x 1 matrix`  
`Boolean B[:, :, :] = fill(false, 0, 1, 0) // a Boolean 0 x 1 x 0 matrix`
- It is not possible to access an element of an empty matrix, e.g. `v[j,k]` is wrong if “`v=[]`” because the assertion fails that the index must be bigger than one.
- Size-requirements of operations, such as `+`, `-`, have also to be fulfilled if a dimension is zero. E.g.  
`Real[3,0] A, B;`  
`Real[0,0] C;`  
`A + B // fine, result is an empty matrix`  
`A + C // error, sizes do not agree`
- Multiplication of two empty matrices results in a zero matrix if the result matrix has no zero dimension sizes, i.e.,  
`Real[0,m]*Real[m,n] = Real[0,n] (empty matrix)`  
`Real[m,n]*Real[n,0] = Real[m,0] (empty matrix)`  
`Real[m,0]*Real[0,n] = zeros(m,n) (non-empty matrix, with zero elements).`

*[Example:*

```

Real u[p], x[n], y[q], A[n,n], B[n,p], C[q,n], D[q,p];
der(x) = A*x + B*u
y = C*x + D*u
```

*Assume n=0, p>0, q>0: Results in "y = D\*u"*

*]*

### 3.2.6 Final element modification

An element defined as **final** in an element modification cannot be modified by a modification or by a redeclaration. All elements of a **final** element are also **final**. *[Setting the value of a parameter in an experiment environment is conceptually treated as a modification. This implies that a final modification equation of a parameter cannot be changed in a simulation environment].*

*[Examples:*

```

type Angle = Real(final quantity="Angle", final unit ="rad",
                     displayUnit="deg");
Angle a1(unit="deg");           // error, since unit declared as final!
Angle a2(displayUnit="rad");    // fine
```

```

model TransferFunction
  parameter Real b[:] = {1} "numerator coefficient vector";
  parameter Real a[:] = {1,1} "denominator coefficient vector";
  ...
end TransferFunction;

model PI "PI controller";
  parameter Real k=1 "gain";
  parameter Real T=1 "time constant";
  TransferFunction tf(final b={T,1}, final a={T,0});
end PI;

model Test
  PI c1(k=2, T=3); // fine
  PI c2(b={1}); // error, b is declared as final
end Test;

```

]

**Note:** In the previous versions of Modelica (Modelica 1.0 and 1.1), the **final** keyword had three different meanings depending on the situation where it was used. To simplify the semantics, in Modelica 1.2, **final** is only used in modifications to prevent further modifications and redeclarations. As a consequence, components have to be explicitly defined as **replaceable**, if they shall be redeclared (previously, this was the default and **final** was used to prevent redeclarations).

### 3.2.7 Short class definition

A class definition of the form

```
class IDENT1 = IDENT2 class_modification ;
```

is identical to the longer form

```
class IDENT1
  extends IDENT2 class_modification ;
end IDENT1;
```

A short class definition of the form

```
type TN = T[N] (optional modifier) ;
```

where N represents arbitrary array dimensions, conceptually yields an array class

```
array TN
  T[n] _ (optional modifiers);
end TN;
```

Such an array class has exactly one anonymous component (\_). When a component of such an array class type is instantiated, the resulting instantiated component type is an array type with the same dimensions as \_ and with the optional modifier applied.

*[Example:*

```
type Force = Real[3](unit={"Nm", "Nm", "Nm"}) ;
Force f1;
Real f2[3](unit={"Nm", "Nm", "Nm"}) ;
```

*the types of f1 and f2 are identical.]*

### 3.2.8 Local class definition

The local class should be statically instantiable with the partially instantiated parent of the local class apart from local class components that are partial or outer. The environment is the modification of any parent class element modification with the same name as the local class, or an empty environment.

The uninstantiated local class together with its environment becomes an element of the instantiated parent class.

*[The following example demonstrates parameterization of a local class:*

```
class C1
  class Voltage = Real(unit="V");
  Voltage v1, v2;
end C1;
class C2
  extends C1(Voltage(unit="kV"));
end C2;
```

*Instantiation of class C2 yields a local class Voltage with unit-modifier "kV". The variables v1 and v2 instantiate this local class and thus have unit "kV".]*

### 3.2.9 Extends clause

The name of the base class is looked up in the partially instantiated parent of the extends clause. The found base class is instantiated with a new environment and the partially instantiated parent of the extends clause. The new environment is the result of merging

- arguments of all parent environments that match names in the instantiated base class
- the modification of a parent element-modification with the same name as the base class
- the optional class modification of the extends clause

in that order.

*[Examples of the three rules are given in the following example:*

```
class A
  parameter Real a, b;
end A;
class B
  extends A(b=3);           // Rule #3
end B;
class C
  extends B(a=1, A(b=2));  // Rules #1 and #2
end C;
```

J

The elements of the instantiated base class become elements of the instantiated parent class.

*[From the example above we get the following instantiated class:*

```
class Cinstance
  parameter Real a=1;
  parameter Real b=2;
end Cinstance;
```

*The ordering of the merging rules ensures that, given classes A and B defined above,*

```
class C2
  B bcomp(b=1, A(b=2));
end C2;
```

*yields an instance with bcomp.b=1, which overrides b=2.]*

The declaration elements of the instantiated base class shall either

- Not already exist in the partially instantiated parent class *[i.e., have different names]* .
- Be identical to any element of the instantiated parent class with the same name and the same level of protection (**public** or **protected**). In this case, the element of the instantiated base class is ignored.

Otherwise the model is incorrect.

[The second rule says that if an element is inherited multiple times, the first inherited element overrides later inherited elements:]

```
class A
  parameter Real a, b;
end A;

class B
  extends A(a=1);
  extends A(b=2);
end B;
```

Class B is well-formed and yields an instantiated object with elements a and b inherited from the first extends clause:

```
class Binstance
  parameter Real a=1;
  parameter Real b;
end Binstance;
```

]

Equations of the instantiated base class that are syntactically equivalent to equations in the instantiated parent class are discarded. [Note: equations that are mathematically equivalent but not syntactically equivalent are not discarded, hence yield an overdetermined system of equations.]

### 3.2.10 Redeclaration

A **redeclare** construct replaces the declaration of an extends clause, local class or component in the modified element with another declaration. The type specified in the redeclaration shall be a subtype of the type in the original declaration.

The element modifications of the redeclaration and the original declaration are merged in the usual way.

[Example:]

```
class A
  parameter Real x;
end A;
class B
  parameter Real x=3.14, y;      // B is a subtype of A
end B;
class C
  replaceable A a(x=1);
end C;
class D
  extends C(redeclare B a(y=2));
end D;
```

which effectively yields a class D2 with the contents

```
class D2
  B a(x=1, y=2);
end D2;
```

]

The following additional constraints apply to redeclarations:

- only classes and components declared as replaceable can be redeclared with a new type and to allow further redeclarations one must use “**redeclare replaceable**”
- a replaceable class used in an extends clause shall only contain public components [otherwise, it cannot be guaranteed that a redeclaration keeps the protected variables of the replaceable default class]

- an element declared as **constant** cannot be redeclared
- an element declared as **parameter** can only be redeclared with **parameter** or **constant**
- an element declared as **discrete** can only be redeclared with **discrete**, **parameter** or **constant**
- a **function** can only be redeclared as **function**
- an element declared as **flow** can only be redeclared with **flow**
- an element declared as not **flow** can only be redeclared without **flow**

Modelica does not allow a protected element to be redeclared as public, or a public element to be redeclared as protected.

Array dimensions may be redeclared.

## 3.3 Equations

### 3.3.1 Equation clause

The instantiated equation is identical to the non-instantiated equation.

Names in an equation shall be found by looking up in the partially instantiated parent of the equation.

Equation equality **=** shall not be used in an algorithm clause. The assignment operator **:=** shall not be used in an equation clause.

### 3.3.2 If clause

If clauses in **equation** sections which do not have exclusively parameter expressions as switching conditions shall have an **else** clause and each branch shall have the *same number of equations*. [If this condition is violated, the single assignment rule would not hold, because the number of equations may change during simulation although the number of unknowns remains the same].

### 3.3.3 For clause

The expression of a for clause shall be a vector expression. It is evaluated once for each for clause, and is evaluated in the scope immediately enclosing the for clause. In an equation section, the expression of a for clause shall be a parameter expression. The loop-variable is in scope inside the loop-construct and shall not be assigned to.

[Example:

```
for i in 1:10 loop           // i takes the values 1,2,3,...,10
  for r in 1.0 : 1.5 : 5.5 loop // r takes the values 1.0, 2.5, 4.0, 5.5
    for i in {1,3,6,7} loop     // i takes the values 1, 3, 6, 7
```

The loop-variable may hide other variables as in the following example. Using another name for the loop-variable is, however, strongly recommended.

```
constant Integer j=4;
Real x[j];
equation
  for j in 1:j loop // The loop-variable j takes the values 1,2,3,4
    x[j]=j; // Uses the loop-variable j
  end for;
]
```

### 3.3.4 When clause

The expression of a when clause shall be a **discrete** Boolean scalar or vector expression. The equations within a when clause are activated when the scalar or any one of the elements of the vector expression becomes true. A when clause shall not be used within a function class.

[Example:

Equations are activated when  $x$  becomes  $> 2$ :

```
when x > 2 then
  y1 = 2*x + y2;
  y2 = sin(x);
end when;
```

Equations are activated when either  $x$  becomes  $> 2$  or  $\text{sample}(0,2)$  becomes true or  $x$  becomes less than 5:

```
when {x > 2, sample(0,2), x < 5} then
  y1 = 2*x + y2;
  y2 = sin(x);
end when;
```

*The equations in a when clause are sorted independently from each other with all other equations.]*

A when clause

```
when {condition1, condition2, ..., conditionN} then
  ...
end when;
```

is equivalent to the following special if-clause, where Boolean  $b[N]$ ; is necessary because we can only apply edge to variables

```
b:={condition1, condition2, ..., conditionN};
if edge(b[1]) or edge(b[2]) or ... edge(b[N]) then
  ...
end if;
```

with “**edge(A)= A and not pre(A)**” and the additional guarantee, that the equations within this special if clause are only evaluated at event instants.

When clauses cannot be nested.

[Example:

The following when clause is invalid:

```
when x > 2 then
  when y1 > 3 then
    y2 = sin(x);
  end when;
end when;
```

]

### 3.3.5 Assert

The expression of an assert clause shall evaluate to true. [The intent is to perform a test of model validity and to report the failed assertion to the user if the expression evaluates to false. The means of reporting a failed assertion are dependent on the simulation environment. The intention is that the current evaluation of the model should stop if when an assert with a false condition is encountered, but the tool should continue the current analysis (e.g. by using a shorter stepsize).]

### 3.3.6 Terminate

The terminate function successfully terminates the analysis which was carried out. The function has a string argument indicating the reason for the success. *[The intention is to give more complex stopping criteria than a fixed point in time. Example:*

```
model ThrowingBall
  Real x(start=0);
  Real y(start=1);
equation
  der(x)=...
  der(y)=...
when y<0 then
  terminate("The ball touches the ground");
end when;
end ThrowingBall;
]
```

### 3.3.7 Connections

Connections between objects are introduced by the **connect** statement in the equation part of a class. The **connect** construct takes two references to connectors, each of which is either an element of the same class as the **connect** statement or an element of one of its components. The two main tasks are to:

- Build connection sets from **connect** statements.
- Generate equations for the complete model.

Definitions:

Connection sets

A connection set is a set of variables connected by means of connect clauses. A connection set shall contain either only flow variables or only non-flow variables.

Inside and outside connectors

In an element instance M, each connector element of M is called an outside connector with respect to M. All other connector elements that are hierarchically inside M, but not in one of the outer connectors of M, is called an inside connector with respect to M.

*[Example: in connect(a,b,c) 'a' is an outer connector and 'b,c' is an inner connector, unless 'b' is a connector.]*

#### 3.3.7.1 Generation of connection equations

Before generating connection equations outer elements are resolved to the corresponding inner elements in the instance hierarchy. The arguments to each connect-statement are resolved to two connector elements, and the connection is moved up zero or more times in the instance hierarchy to the first element instance that both the connectors are hierarchically contained in it.

For every use of the connect statement

```
connect(a, b);
```

the primitive components of a and b form a connection set. If any of them already occur in a connection set from previous connections with matching inside/outside, these sets are merged to form one connection set.

Composite connector types are broken down into primitive components. Each connection set is used to generate equations for across and through (zero-sum) variables of the form

```
a1 = a2 = ... = an;
z1 + z2 + (-z3) + ... + zn = 0;
```

In order to generate equations for through variables [*using the flow prefix*], the sign used for the connector variable  $z_i$  above is +1 for inside connectors and -1 for outside connectors [ $z_3$  in the example above].

For each flow (zero-sum) variable in a connector that is not connected as an inside connector in any element instance the following equation is implicitly generated:

```
z = 0;
```

The bold-face **0** represents an array or scalar zero of appropriate dimensions (i.e. the same size as  $z$ ).

### 3.3.7.2 Restrictions

A component of a connector declared with the **input** type prefix shall not occur as inside connector in more than one **connect** statement. A component of a connector declared with the **output** type prefix shall not occur as outside connector in more than one **connect** statement. If two components declared with the **input** type prefix are connected in a **connect** statement one must be an inside connector and the other an outside connector. If two components declared with the **output** type prefix are connected in a **connect** statement one must be an inside connector and the other an outside connector.

Subscripts in a connector reference shall be constant expressions.

If the array sizes do not match, the original variables are filled with one-sized dimensions from the left until the number of dimensions match before the connection set equations are generated.

Constants or parameters in connected components yield the appropriate assert statements; connections are not generated.

## 3.4 Functions

There are two forms of function application, see section 2.2.7. In the first form,

```
f(3.5, 5.76)
```

the arguments are associated with the [*formal*] parameters according to their position in the argument list. Thus argument  $i$  is passed to parameter  $i$ , where the order of the parameters is given by the order of the component declarations in the function definition. The first input component is parameter number 1, the second input component is parameter number 2, and so on. When a function is called in this way, the number of arguments and parameters must be the same.

In the second form of function application,

```
g(x=3.5, y=5.76)
```

the parameters are explicitly associated with the arguments by means of equations in the argument list. Parameters that have default values need not be specified in the argument list.

The type of each argument must agree with the type of the corresponding parameter, except where the standard type coercions can be used to make the types agree. (See also section 3.2.2.5 on applying scalar functions to arrays.)

[*Example. Suppose a function f is defined as follows:*

```
function f
  input Real x;
  input Real y;
  input Real z := 10.0;
  output Real r;
  ...
```

```
end f;
```

Then the following two applications are equivalent:

```
f(1.0, 2.0, 10.0)
f(y = 2.0, x = 1.0)
```

```
]
```

A function may have more than one output component, corresponding to multiple return values. When a function has a single return value, a function application is an expression whose value and type are given by the value and type of the output component.

The only way to call a function having more than one output component is to make the function call the RHS of an equation or assignment. In these cases, the LHS of the equation or assignment must be a list of component references within parentheses. The component references are associated with the output components according to their position in the list. Thus output component *i* is set equal to, or assigned to, component reference *i* in the list, where the order of the output components is given by the order of the component declarations in the function definition.

The number of component references in the list must agree with the number of output components.

The type of each output parameter must agree with the type of the corresponding component references in the list on the LHS.

[Example. Suppose a function *f* is defined as follows:

```
function f
  input Real x;
  input Real y;
  output Real r1;
  output Real r2;
  output Real r3;
  ...
end f;
```

Then the following equation and assignment show the two possible ways of calling *f*:

```
(x, y, z) = f(1.0, 2.0);
(x, y, z) := f(1.0, 2.0);
```

```
]
```

The only permissible use of an expression in the form of a list of expressions in parentheses, is when it is used as the LHS of an equation or assignment where the RHS is an application of a function with more than one output component. In this case, the expressions in the list shall be component references.

[Example. The following are illegal:

```
(x+1, 3.0, z/y) = f(1.0, 2.0); // Not a list of component references.
(x, y, z) + (u, v, w)           // Not LHS of suitable eqn/assignment.
```

```
]
```

## 3.5 Evaluation

A tool is free to solve equations, reorder expressions and to not evaluate expressions if their values do not influence the result (e.g. short-circuit evaluation of boolean expressions). If-statements and if-expressions guarantee that their clauses are only evaluated if the appropriate condition is true, but relational operators generating state or time events will during continuous integration have the value from the most recent event.

[Example. If one wants to guard an expression against evaluation it should be guarded by an if

```
Boolean v[n];
```

```

Boolean b;
Integer I;
equation
  x=v[I] and (I>=1 and I<=n); // Invalid
  x=if (I>=1 and I<=n) then v[I] else false; // Correct

```

To guard square against square root of negative number use noEvent:

```

der(h)=if h>0 then -c*sqrt(h) else 0; // Incorrect
der(h)=if noEvent(h>0) then -c*sqrt(h) else 0; // Correct
]
```

## 3.6 Events and Synchronization

The integration is halted and an event occurs whenever a Real elementary relation, e.g. “ $x > 2$ ”, changes its value. The value of a relation can only be changed at event instants [*in other words, Real elementary relations induce state or time events*]. The relation which triggered an event changes its value when evaluated literally before the model is processed at the event instant [*in other words, a root finding mechanism is needed which determines a small time interval in which the relation changes its value; the event occurs at the right side of this interval*]. At an event instant, a relation is taken literally. During continuous integration a Real elementary relation has the *constant* value of the relation from the last event instant.

[Example:

```
Y = if u > uMax then uMax else if u < uMin then uMin else u;
```

During continuous integration always the same if branch is evaluated. The integration is halted whenever  $u$ - $uMax$  or  $u$ - $uMin$  crosses zero. At the event instant, the correct if-branch is selected and the integration is restarted.

Numerical integration methods of order  $n$  ( $n \geq 1$ ) require continuous model equations which are differentiable upto order  $n$ . This requirement can be fulfilled if Real elementary relations are not treated literally but as defined above, because discontinuous changes can only occur at event instants and no longer during continuous integration.]

[It is a quality of implementation issue that the following special relations

```
time >= discrete expression
time < discrete expression
```

trigger a time event at “ $time = discrete expression$ ”, i.e., the event instant is known in advance and no iteration is needed to find the exact event instant.]

Relations are taken literally also during continuous integration, if the relation or the expression in which the relation is present, are the argument of the **noEvent(..)** function. The noEvent feature is propagated to all subrelations in the scope of the noEvent function.

[Example:

```
Y = noEvent( if u > uMax then uMax else if u < uMin then uMin else u);
```

The if-expression is taken literally without inducing state events.

The **noEvent** function is useful, if e.g. the modeller can guarantee that the used if-clauses fulfill at least the continuity requirement of integrators. In this case the simulation speed is improved, since no state event iterations occur during integration. Furthermore, the **noEvent** function is used to guard against “outside domain” errors, e.g.  $y = \text{if } \text{noEvent}(x \geq 0) \text{ then } \sqrt{x} \text{ else } 0$ .]

All equations and assignment statements within *when clauses* and all assignment statements within *function classes* are implicitly treated with the **noEvent** function, i.e., relations within the scope of these operators never induce state or time events. [Using state events in when-clauses is unnecessary because the body of a when

*clause is not evaluated during continuous integration.]*

*[Example:*

```
Limit1 = noEvent(x1 > 1);
  Limit2 = x2 > 10;
  Limit = Limit1 or Limit2;
  when Limit then           // error, Limit is no discrete expression
    Close = true;
  end when;
```

*This is an error, because Limit1 may change during continuous integration, i.e., it is not a discrete-time variable and therefore the when condition Limit is also no discrete-time variable (when conditions need to be discrete expressions in order to guarantee that when-equations are never evaluated during continuous integration).*

Modelica is based on the synchronous data flow principle which is defined in the following way:

1. All variables keep their actual values until these values are explicitly changed. Variable values can be accessed at any time instant during continuous integration and at event instants.
2. At every time instant, during continuous integration and at event instants, the active equations express relations between variables which have to be fulfilled *concurrently* (equations are not active if the corresponding if-branch, when-clause or block in which the equation is present is not active).
3. Computation and communication at an event instant does not take time. *[If computation or communication time has to be simulated, this property has to be explicitly modeled].*
4. The total number of equations is identical to the total number of unknown variables (= single assignment rule).

*[These rules guarantee that variables are always defined by a unique set of equations. It is not possible that a variable is e.g. defined by two equations, which would give rise to conflicts or non-deterministic behaviour. Furthermore, the continuous and the discrete parts of a model are always automatically “synchronized”.*

*Example:*

```
when condition1 then
  close = true;
end when;

when condition2 then
  close = false;
end when;
```

*This is not a valid model because rule 4 is violated since there are two equations for the single unknown variable close. If this would be a valid model, a conflict occurs when both conditions become true at the same time instant, since no priorities between the two equations are assigned. To become valid, the model has to be changed to:*

```
when {condition1, condition2} then
  close = if edge(condition1) then true else false;
end when;
```

*Here, it is well-defined if both conditions become true at the same time instant (condition1 has a higher priority than condition2).]*

There is no guarantee that two different events occur at the same time instant.

*[As a consequence, synchronization of events has to be explicitly programmed in the model, e.g. via counters.*

*Example:*

```
Boolean fastSample, slowSample;
Integer ticks(start=0);

equation
  fastSample = sample(0,1);
  when fastSample then
```

```

  ticks      = if pre(ticks) < 5 then pre(ticks)+1 else 0;
  slowSample = pre(ticks) == 0;
end when;

when fastSample then    // fast sampling
  ...
end when;

when slowSample then    // slow sampling (5-times slower)
  ...
end when;

```

*The slowSample when-clause is evaluated at every 5<sup>th</sup> occurrence of the fastSample when clause.]*

*[The single assignment rule and the requirement to explicitly program the synchronization of events allow a certain degree of model verification already at compile time. For example, “deadlock” between different when-clauses is present if there are algebraic loops between the equations of the when-clauses.]*

## 3.7 Restricted classes

The keyword **class** can be replaced by one of the following keywords: **record**, **type**, **connector**, **model**, **block**, **package** or **function**. Certain restrictions will then be imposed on the content of such a definition. The following table summarizes the restrictions.

<b>record</b>	No equations are allowed in the definition or in any of its components. May not be used in connections.
<b>type</b>	May only be extension to the predefined types, records or array of type.
<b>connector</b>	No equations are allowed in the definition or in any of its components.
<b>model</b>	May not be used in connections.
<b>block</b>	Fixed causality, input-output block. Each component of an interface must either have Causality equal to Input or Output. May not be used in connections.
<b>package</b>	May only contain declarations of classes and constants.
<b>function</b>	Same restrictions as for block. Additional restrictions: no equations, at most one algorithm section. Calling a function requires either an algorithm section or an external function interface. A function can not contain calls to the Modelica built-in operators <b>der</b> , <b>initial</b> , <b>terminal</b> , <b>sample</b> , <b>pre</b> , <b>edge</b> , <b>change</b> , <b>reinit</b> , <b>delay</b> and <b>cardinality</b> .

## 3.8 Variable attributes

The attributes of the predefined variable types are described below with Modelica syntax although they are predefined; redeclaration of any of these types is an error. The definitions use **RealType**, **IntegerType**, **BooleanType** and **StringType** as mnemonics corresponding to machine representations. *[Hence the only way to declare a subtype of e.g. Real is to use the extends mechanism.]*

```

type Real
  RealType value;                                // Accessed without dot-notation
  parameter StringType quantity = "";
  parameter StringType unit = ""    "Unit used in equations";
  parameter StringType displayUnit = ""  "Default display unit";
  parameter RealType min=-Inf, max=+Inf; // Inf denotes a large value
  parameter RealType start = 0;           // Initial value
  parameter BooleanType fixed = true, // default for parameter/constant;
                                             = false; // default for other variables
  parameter RealType nominal;           // Nominal value
equation
  assert(value >= min and value <= max, "Variable value out of limit");

```

```

assert(nominal >= min and nominal <= max, "Nominal value out of limit");
end Real;

type Integer
  IntegerType value;                                // Accessed without dot-notation
  parameter IntegerType min=-Inf, max=+Inf;
  parameter IntegerType start = 0;      // Initial value
  parameter BooleanType fixed = true, // default for parameter/constant;
                                         = false; // default for other variables
equation
  assert(value = min and value <= max, "Variable value out of limit");
end Integer;

type Boolean
  BooleanType value;                                // Accessed without dot-notation
  parameter BooleanType start = false; // Initial value
  parameter BooleanType fixed = true, // default for parameter/constant;
                                         = false, // default for other variables
end Boolean;

type String
  StringType value;                                // Accessed without dot-notation
  parameter StringType start = "";      // Initial value
end String;

```

The attributes “start” and “fixed” define the initial conditions for a variable for analysisType = “static”. “fixed=false” means an initial guess, i.e., value may be changed by static analyzer. “fixed=true” means a required value. Before other analysisTypes (such as “dynamic”) are performed, the analysisType “static” has to be carried out first. The resulting consistent set of values for ALL model variables is used as initial values for the analysis to be performed.

The attribute “nominal” gives the nominal value for the variable. The user need not set it even though the standard does not define a default value. *[The nominal value can be used by an analysis tool to determine appropriate tolerances or epsilons, or may be used for scaling. For example, the absolute tolerance for an integrator could be computed as “absTol = abs(nominal)\*relTol/100”. A default value is not provided in order that in cases such as “a=b”, where “b” has a nominal value but not “a”, the nominal value can be propagated to the other variable.]* *[For external functions in C, RealType by default maps to double and IntegerType by default maps to int. In the mapping proposed in Annex F of the future C9X standard, RealType/double matches the IEC 60559:1989 (ANSI/IEEE 754-1985) double format. Typically IntegerType represents a 32-bit 2-complement signed integer.]*

## 3.9 Intrinsic library functionality

### 3.9.1 Built-in variable time

All declared variables are functions of the independent variable **time**. Time is a built-in variable available in all classes, which is treated as an input variable. It is implicitly defined as:

```

input Real time (final quantity = "Time",
  final unit      = "s");

```

The value of the **start** attribute of time is set to the time instant at which the simulation is started.

*[Example:*

Trigger an event at start time + 10 s:

```

parameter Real T0 = time.start + 10;
when time >= T0 then

```

```

  ...
end when;
]

```

### 3.9.2 Modelica built-in operators

Built-in operators of Modelica have the same syntax as a function call. However, they do **not** behave as a mathematical function, because the result depends not only on the input arguments but also on the status of the simulation. The following operators are supported:

<b>der(x)</b>	The time derivative of x. Variable x need to be a (non-discrete) subtype of Real. If x is an array, the operator is applied to all elements of the array.
<b>analysisType()</b>	Returns the most appropriate analysis type for the context in which the model is used. The analysis type is returned as a string. The following return values are predefined: "dynamic": Solve initial value problem "static": Solve "static" problem where all derivatives are constant and time is fixed (e.g. trimming, equilibrium analysis) "linear": Transform continuous part of model in a linear system.
<b>initial()</b>	Returns true at the beginning of analysis (where time is equal to time.start).
<b>terminal()</b>	Returns true at the end of a successful analysis.
<b>noEvent(expr)</b>	Real elementary relations within expr are taken literally, i.e., no state or time event is triggered.
<b>sample(start,interval)</b>	Returns true and triggers time events at time instants "start + i*interval" (i=0,1,...). During continuous integration the operator returns always false. The starting time "start" and the sample interval "interval" need to be parameter expressions and need to be a subtype of Real or Integer.
<b>pre(y)</b>	Returns the "left limit" $y(t^{pre})$ of variable y(t) at a time instant t. At an event instant, $y(t^{pre})$ is the value of y after the last event iteration at time instant t (see comment below). The <b>pre</b> operator can be applied if the following three conditions are fulfilled simultaneously: (a) variable y is a subtype of Boolean, Integer or Real, (b) the operator is applied in a <b>when</b> body or y is declared as <b>discrete</b> , (c) the operator is <b>not</b> applied in a <b>function</b> class. At the initial time $pre(y) = y.start$ , i.e., the left limit of y is identical to the start value.
<b>edge(b)</b>	Is expanded into "(b <b>and</b> <b>not</b> pre(b))" for Boolean variable b. The same restrictions as for the <b>pre</b> operator apply (e.g. not to be used in function classes).
<b>change(v)</b>	Is expanded into "(v <> pre(v))". The same restrictions as for the pre() operator apply.
<b>reinit(x, expr)</b>	Reinitializes state variable x with expr at an event instant. Argument x need to be (a) a subtype of Real and (b) the <b>der</b> -operator need to be applied to it. expr need to be an Integer or Real expression. The reinit operator can only be applied once for the same variable x.
<b>abs(v)</b>	Is expanded into "(if v >= 0 then v else -v)". Argument v needs to be an Integer or Real expression. [Note, outside of a when clause state events are triggered].
<b>sign(v)</b>	Is expanded into "(if v > 0 then 1 else if v < 0 then -1 else 0)". Argument v needs to be an Integer or Real expression. [Note, outside of a when clause state events are triggered]

<b>sqrt(v)</b>	Returns the square root of v if $v \geq 0$ , otherwise an error occurs. Argument v needs to be an Integer or Real expression.
<b>div(x,y)</b>	Returns the algebraic quotient $x/y$ with any fractional part discarded (also known as truncation toward zero). Result and arguments shall have type Real. [Note: this is defined for / in C9X; in Standard C the result for negative numbers is implementation-defined, so the standard function div() must be used.]. The input arguments need to be discrete expressions.
<b>rem(x,y)</b>	Returns the integer remainder of $x/y$ , such that $\text{div}(x,y) * y + \text{rem}(x,y) = x$ . Result and arguments shall have type Real. The input arguments need to be discrete expressions.
<b>ceil(x)</b>	Returns the smallest integer not less than x. Result and argument shall have type Real. The input argument needs to be a discrete expression.
<b>floor(x)</b>	Returns the largest integer not greater than x. Result and argument shall have type Real. The input argument needs to be a discrete expression.
<b>integer(x)</b>	Returns the largest integer not greater than x. The argument shall have type Real. The result has type Integer. The input argument needs to be a discrete expression.
<b>delay(expr,delayTime,delayMax)</b> <b>delay(expr,delayTime)</b>	Returns "expr(time - delayTime)" for $time > time.start + delayTime$ and "expr(time.start)" for $time \leq time.start + delayTime$ . The arguments, i.e., expr, delayTime and delayMax, need to be subtypes of Real. DelayMax needs to be additionally a parameter expression. The following relation shall hold: $0 \leq delayTime \leq delayMax$ , otherwise an error occurs. If delayMax is not supplied in the argument list, delayTime need to be a parameter expression.
<b>cardinality(c)</b>	Returns the number of (internal and external) occurrences of connector instance c in a connect statement as an Integer number.

A new event is triggered if at least for one variable v “**pre(v)  $\leftrightarrow$  v**” after the active model equations are evaluated at an event instant. In this case the model is at once reevaluated. This evaluation sequence is called “*event iteration*”. The integration is restarted, if for all v used in **pre**-operators the following condition holds: “**pre(v) == v**”.

*[If v and pre(v) are only used in when clauses, the translator might mask event iteration for variable v since v cannot change during event iteration. It is a “quality of implementation” to find the minimal loops for event iteration, i.e., not all parts of the model need to be reevaluated.]*

*The language allows that mixed algebraic systems of equations occur where the unknown variables are of type Real, Integer or Boolean. These systems of equations can be solved by a global fix point iteration scheme, similarly to the event iteration, by fixing the Boolean and Integer unknowns during one iteration. Again, it is a quality of implementation to solve these systems more efficiently, e.g., by applying the fix point iteration scheme to a subset of the model equations.]*

The **reinit** operator does not break the single assignment rule, because **reinit(x,expr)** makes the previously known state variable x unknown and introduces the equation “ $x = \text{expr}$ ”.

*[If a higher index system is present, i.e. constraints between state variables, some state variables need to be redefined to non-state variables. If possible, non-state variables should be chosen in such a way that states with an applied **reinit** operator are not utilized. If this is not possible, an error occurs, because the reinit operator is applied on a non-state variable.]*

*Examples for the usage of the **reinit** operator:*

*Bouncing ball:*

```

der(h) = v;
der(v) = -g;
when h < 0 then
  reinit(v, -e*v);
end when;

```

*Self-initializing block:*

```

block PT1 " first order filter"
  parameter Real T "time constant ";
  parameter Real k "gain";
  input Real u;
  output Real y;
protected
  Real x;
equation
  der(x) = (u - x) / T;
  y = k*x;
when initial() then
  reinit(x, u); // initialize, such that der(x) = 0.
end when
end PT1;

model Test
  PT1 b1, b2, b3;
  input u;
equation
  b1.u = u;
  connect(b1.y, b2.u);
  connect(b2.y, b3.u);
end Test;

```

*Given the input signal u, all 3 blocks b1, b2, b3 are initialized at their stationary value.]*

*[The **abs** and **sign** operator trigger state events if used outside of a **when** clause. If this is not desired, the **noEvent** function can be applied to them. E.g. **noEvent(abs(v))** is  $|v|$ ]*

*The **div**, **rem**, **ceil**, **floor**, **integer** operators require discrete expressions as input arguments, i.e., these functions can be either called in **when** clauses or the input arguments need to be **discrete** variables. The reason for this restriction is that these operators are not differentiable, i.e., the partial derivatives of the result with respect to the input arguments are no continuous functions. Since this is a pre-requisite for continuous integration, it must be guaranteed that these operators are not called during continuous integration or if they are called, produce always the same result.*

*The **delay** operator allows a numerical sound implementation by interpolating in the (internal) integrator polynomials, as well as a more simple realization by interpolating linearly in a buffer containing past values of expression **expr**. Without further information, the complete time history of the delayed signals need to be stored, because the delay time may change during simulation. To avoid excessive storage requirements and to enhance efficiency, the maximum allowed delay time has to be given via **delayMax**. This gives an upper bound on the values of the delayed signals which have to be stored. For realtime simulation where fixed step size integrators are used, this information is sufficient to allocate the necessary storage for the internal buffer before the simulation starts. For variable step size integrators, the buffer size is dynamic during integration. In principle, a delay operator could break algebraic loops. For simplicity, this is not supported because the minimum delay time has to be given as additional argument to be fixed at compile time. Furthermore, the maximum step size of the integrator is limited by this minimum delay time in order to avoid extrapolation in the delay buffer.*

*The **cardinality** operator allows the definition of connection dependent equations in a model, for example:*

```
connector Pin
```

```
Real      v;
flow Real i;
end Pin;

model Resistor
  Pin p, n;
equation
  // Handle cases if pins are not connected
  if cardinality(p) == 0 and cardinality(n) == 0 then
    p.v = 0; n.v = 0;
  else if cardinality(p) == 0 then
    p.i = 0;
  else if cardinality(n) == 0 then
    n.i = 0;
  end if

  // Equations of resistor
  ...
end Resistor;
```

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## 4 Mathematical description of Hybrid DAEs

In this section, the mapping of a Modelica model into an appropriate mathematical description form is discussed.

The result of the modeling process is a set of ordinary differential equations, often accompanied with algebraic constraint equations, thus forming a set of Differential and Algebraic Equations (DAE). The initial values of the state variables need to be specified, implying that the DAE is mathematically formulated as a so-called Initial Value Problem. This DAE is used for simulation or other analysis activities. DAEs may have discontinuities or the structure of a DAE may change at certain points in time. Such types of DAEs are called *hybrid DAEs*. Events are used to stop continuous integration at discontinuities of a hybrid DAE. After applying the discontinuous change, the integration is restarted. A hybrid DAE is mathematically described by a set of equations of the form

- (1a) Residue Equations:  $0 = \mathbf{f}(\mathbf{dx}/dt, \mathbf{x}, \mathbf{y}, t, \mathbf{m})$ ,  $d\mathbf{f}/[d\mathbf{x}/dt; \mathbf{y}]$  is regular
- (1b) Monitor Functions:  $\mathbf{z} := \mathbf{g}(\mathbf{dx}/dt, \mathbf{x}, \mathbf{y}, t, \mathbf{m})$
- (1c) Update Equations :  $0 = \mathbf{h}(\mathbf{dx}/dt, \mathbf{x}^{\text{known}}, \mathbf{x}^{\text{reinit}}, \mathbf{y}, t, \mathbf{m}, \text{pre}(\mathbf{m}))$

Additionally, every equation is a function of the parameters **p** and of the input functions **u(t)**. This dependency is not explicitly shown in (1) for clarity of the equations. The variables have the following meaning:

**t** time, the independent (real) variable.

**x(t)** (Real) variables appearing differentiated ( $\mathbf{x}^{\text{known}}$  is the part of **x** which is always known;  $\mathbf{x}^{\text{reinit}}$  is the other part of **x** which is reinitialized at an event instant).

**y(t)** (Real) algebraic variables.

**u(t)** known (Real) functions of time.

**m** discrete variables of type Real, Boolean or Integer defining the current **mode**.

**pre(m)** are the values of **m** immediately before the current event occurred.

**p** parameters, i.e., variables without any time-dependency.

The *residue equations* (1a) are used for continuous integration. During integration, the discrete variables **m** are not changed. The *monitor functions* (1b) are also evaluated during continuous integration. If one of the signals **z** crosses zero, the integration is halted and an event occurs. The special case of a time event, " $z = t - t_e$ ", is also included. For efficiency reasons, time events are usually treated in a special way, since the time instant of such an event is known in advance. At every event instant, the *update functions* (1c) are used to determine new values of the discrete variables and of new initial values for the states **x**. The change of discrete variables may characterize a new structure of a DAE where elements of the state vector **x** are *disabled*. In other words, the number of state variables, algebraic variables and residue equations of a DAE may change at event instants by disabling the appropriate part of the DAE. For clarity of the equations, this is not explicitly shown by an additional index in (1).

At an event instant, including the initial event, the model equations are reinitialized according to the following iteration procedure:

```

loop
  solve
     $0 = \mathbf{f}(\mathbf{dx}/dt, \mathbf{x}^{\text{known}}, \mathbf{x}^{\text{reinit}}, \mathbf{y}, t, \mathbf{m})$ 
     $0 = \mathbf{h}(\mathbf{dx}/dt, \mathbf{x}^{\text{known}}, \mathbf{x}^{\text{reinit}}, \mathbf{y}, t, \mathbf{m}, \text{pre}(\mathbf{m}))$ 
  for  $\mathbf{dx}/dt, \mathbf{x}^{\text{reinit}}, \mathbf{y}, \mathbf{m}$ , where  $\mathbf{x}^{\text{known}}, t, \text{pre}(\mathbf{m})$  are fixed
  if  $\mathbf{m} == \text{pre}(\mathbf{m})$  then break
   $\text{pre}(\mathbf{m}) := \mathbf{m}$ 

```

```
  end loop
```

At every iteration a set of mixed Real, Boolean, Integer equations (1a),(1c) has to be solved for the indicated variables. In order that this is possible, at least the Jacobian of (1a) needs to be regular, as it was stated in (1a). This set of equations can e.g. be solved by a global fixed point iteration scheme which can be combined with the event iteration:

```
loop
  solve "0 = f(dx/dt, xknown, xreinit, y, t, m)"
    for dx/dt, y, where xknown, xreinit, t, m are fixed
  solve "0 = h(dx/dt, xknown, xreinit, y, t, m, pre(m))"
    for xreinit, m where dx/dt, y, xknown, t, pre(m) are fixed
  if m == pre(m) then break
  pre(m) := m
end loop
```

The hybrid DAE (1) is not the most general one, but it has a clearly defined view and structure. Especially, (1a) can be transformed into state space form, at least numerically, since the Jacobian is required to be regular.

Generalizations are possible in the direction of higher index DAEs where the Jacobian of (1a) is singular. This leads to additional difficulties during integration and especially for event restart because the non-linear equation cannot be solved due to the singular Jacobian. Other generalizations concern the determination of the initial configuration by allowing the specification of any variable at the initial time and by calculating the remaining ones. A third generalization may use other algorithms to determine a consistent configuration after an event occurred, e.g., by solving a complementary problem, see (Pfeiffer and Glocker 1996) for details. For a *certain class* of higher index DAE systems, algorithms are available to automatically differentiate selected equations of (1a), choose appropriate variables to be no longer states (= dummy derivative method) and transform to a DAE (1) with a regular Jacobian.

The Modelica language allows a direct and convenient specification of physical systems. A *Modelica translator maps a Modelica model into a hybrid DAE (1)*, or in one of its generalizations if these are available. The mapping into (1) is straightforward by expanding all class definitions (flattening the inheritance tree) and adding the equations and assignment statements of the expanded classes for every instance of the model to (1). The resulting hybrid DAE usually contains a huge number of sparse equations. Therefore, direct simulation of a hybrid DAE (1) which was generated by a Modelica translator requires sparse matrix methods.

There are several simulation environments available, such as Allan, Dymola, gPROMS, Ida (NMF) or Omola, which preprocess (1) symbolically to arrive at a form which can be evaluated more efficiently by numerical algorithms. Especially, efficient graph-theoretical algorithms are available to transform (1) automatically into the following form which is called **sorted** hybrid DAE:

```
(2a) Residue Equations: 0 = fx(dxi/dt, yi, x, t, m),
(2b) Exp. dx-Functions: dxe/dt := fx(dxi/dt, yi, x, t, m)
(2c) Exp. y-Functions : ye := fy(dxi/dt, yi, x, t, m)
(2d) Monitor Functions: z := g(dxi/dt, yi, x, t, m)
(2e) Update Equations : [m, xreinit] := h(dxi/dt, yi, xknown, t, m, pre(m))
```

where the vector of algebraic variables  $y$  is split into implicit variables  $y^i$  and explicitly solvable algebraic variables  $y^e$ . The vector of state derivatives  $dx/dt$  is split into implicit variables  $dx^i/dt$  and explicitly solvable variables  $dx^e/dt$ , respectively. When using an *implicit integrator*, only equations (2a,2b) need to be solved during continuous integration. Equations (2c) are effectively hidden from the solver. They need only be evaluated for external usage (e.g., to store output points to be plotted). At initial time and at events, the non-linear equation of reduced dimension (2a) has to be solved. Again the dimension of the original equations has reduced considerably. It is also possible to use *explicit integration* methods, such as Runge-Kutta algorithms. During continuous integration, the integrator provides  $x$  and  $t$ . The model function solves (2a) for the implicit variables, uses the result to evaluate (2b) and returns the complete vector of state derivatives  $dx/dt$ . This procedure is useful for *real-time* simulation where only explicit one-step methods can be used and for non-stiff systems where the number of implicit equations is small and/or linear.

To summarize, a Modelica translator maps a Modelica model into the hybrid DAE (1). By a subsequent symbolic processing, (1) can be transformed into the sorted hybrid DAE (2).

---

# 5 Unit expressions

---

Unless otherwise stated, the syntax and semantics of unit expressions in Modelica are conform with the international standards ISO 31/0-1992 "General principles concerning quantities, units and symbols" and ISO 1000-1992 "SI units and recommendations for the use of their multiples and of certain other units". Unfortunately, neither these two standards nor other existing or emerging ISO standards define a formal syntax for unit expressions. There are recommendations and Modelica exploits them.

Examples for the syntax of unit expressions used in Modelica: "N.m", "kg.m/s<sup>2</sup>", "kg.m.s<sup>-2</sup>" "1/rad", "mm/s".

## 5.1 The Syntax of unit expressions

```
unit_expression:
  unit_numerator [ "/" unit_denominator ]

unit_numerator:
  "1" | unit_factors | "( " unit_expression " )"

unit_denominator:
  unit_factor | "( " unit_expression " )"

The unit of measure of a dimension free quantity is denoted by "1". The ISO standard does not define any precedence between multiplications and divisions. The ISO recommendation is to have at most one division, where the expression to the right of "/" either contains no multiplications or is enclosed within parentheses. It is also possible to use negative exponents, for example, "J/(kg.K)" may be written as "J.kg-1.K-1".
```

```
unit_factors:
  unit_factor [ unit_mulop unit_factors ]
```

```
unit_mulop:
```

```
  "
```

The ISO standard allows that a multiplication operator symbol is left out. However, Modelica enforces the ISO recommendation that each multiplication operator is explicitly written out in formal specifications. For example, Modelica does not support "Nm" for newtonmeter, but requires it to be written as "N.m".

The preferred ISO symbol for the multiplication operator is a "dot" a bit above the base line: "·". Modelica supports the ISO alternative "·", which is an ordinary "dot" on the base line.

```
unit_factor:
  unit_operand [ unit_exponent ]
```

```
unit_exponent:
```

```
  [ "+" | "-" ] integer
```

The ISO standard does not define any operator symbol for exponentiation. A unit\_factor consists of a unit\_operand possibly suffixed by a possibly signed integer number, which is interpreted as an exponent. There must be no spacing between the unit\_operand and a possible unit\_exponent.

```
unit_operand:
  unit_symbol | unit_prefix unit_symbol
```

```
unit_prefix:
```

```
  Y | Z | E | P | T | G | M | k | h | da | d | c | m | u | p | f | a | z |
```

```
  y
```

A unit\_symbol is a string of letters. A basic support of units in Modelica should know the basic and derived units of the SI system. It is possible to support user defined unit symbols. In the base version Greek letters is not supported, but full names must then be written, for example "Ohm".

A unit\_operand should first be interpreted as a unit\_symbol and only if not successful the second alternative assuming a prefixed operand should be exploited. There must be no spacing between the unit\_symbol and a possible unit\_prefix. The value of the prefixes are according to the ISO standard. The letter "u" is used as a symbol for the prefix micro.

## 5.2 Examples

- The unit expression "m" means meter and not milli ( $10^{-3}$ ), since prefixes cannot be used in isolation. For millimeter use "mm" and for squaremeter,  $m^2$ , write "m2".
- The expression "mm2" means  $mm^2 = (10^{-3}m)^2 = 10^{-6}m^2$ . Note that exponentiation includes the prefix. The unit expression "T" means Tesla, but note that the letter "T" is also the symbol for the prefix tera which has a multiplier value of  $10^{12}$ .

# 6 External function interface

---

## 6.1 Overview

Here, the word **function** is used to refer to an arbitrary external routine, whether or not the routine has a return value or returns its result via output parameters (or both). The Modelica external function call interface provides the following:

- Support for external functions written in C and FORTRAN 77. Other languages, e.g. C++ and Fortran 90, may be supported in the future.
- Mapping of argument types from Modelica to the target language and back.
- Natural type conversion rules in the sense that there is a mapping from Modelica to standard libraries of the target language.
- Handling arbitrary parameter order for the external function.
- Passing arrays to and from external functions where the dimension sizes are passed as explicit integer parameters.
- Handling of external function parameters which are used both for input and output.

The format of an external function declaration is as follows.

```
function IDENT string_comment
  { component_clause ";" }
  [ protected { component_clause ";" } ]
  external [ language_specification ] [ external_function_call ] ";" 
  [ annotation ";" ]
end IDENT;
```

Components in the public part of an external function declaration shall be declared either as **input** or **output**.  
*[This is just as for any other function. The components in the protected part allows local variables for temporary storage to be declared.]*

The *language-specification* must currently be one of "C" or "FORTRAN 77". Unless the external language is specified, it is assumed to be C.

The *external-function-call* specification allows functions whose prototypes do not match the default assumptions as defined below to be called. It also gives the name used to call the external function. If the external call is not given explicitly, this name is assumed to be the same as the Modelica name.

The only permissible kinds of expressions in the argument list are identifiers, scalar constants, and the function **size** applied to an array and a constant dimension number. The annotations are used to pass additional information to the compiler when necessary. Currently, the only supported annotation is **arrayLayout**, which can be either "rowMajor" or "columnMajor".

## 6.2 Argument type mapping

The arguments of the external function are declared in the same order as in the Modelica declaration, unless specified otherwise in an explicit external function call. Protected variables (i.e. temporaries) are passed in the same way as outputs, whereas constants and size-expression are passed as inputs.

## 6.2.1 Simple types

Arguments of **simple** types are by default mapped as follows for C:

Modelica	C	
	Input	Output
Real	double	double *
Integer	int	int *
Boolean	int	int *
String	const char *	Not allowed.

An exception is made when the argument is of the form `size(..., ...)`. In this case the corresponding C-type is `size_t`.

Strings are NUL-terminated to facilitate calling of C functions. Currently, returning strings from external C-functions is not supported.

Arguments of **simple** types are by default mapped as follows for FORTRAN 77:

Modelica	FORTRAN 77	
	Input	Output
Real	DOUBLE PRECISION	DOUBLE PRECISION
Integer	INTEGER	INTEGER
Boolean	LOGICAL	LOGICAL

Passing strings to FORTRAN 77 subroutines/functions is currently not supported.

## 6.2.2 Arrays

Unless an explicit function call is present in the external declaration, an arrays is passed by its address followed by  $n$  arguments of type `size_t` with the corresponding array dimension sizes, where  $n$  is the number of dimensions. [*The type `size_t` is a C unsigned integer type.*]

Arrays are by default stored in row-major order when calling C functions and in column-major order when calling FORTRAN 77 functions. These defaults can be overridden by the array layout annotation. See the example below.

The table below shows the mapping of an array argument in the absence of an explicit external function call when calling a C function. The type  $T$  is allowed to be any of the simple types which can be passed to C as defined in section 6.2.1 or a record type as defined in section 6.2.3 and it is mapped to the type  $T'$  as defined in these sections.

Modelica	C	
	Input and Output	
$T[dim_1]$	$T' *, size_t dim_1$	
$T[dim_1, dim_2]$	$T' *, size_t dim_1, size_t dim_2$	
$T[dim_1, ..., dim_n]$	$T' *, size_t dim_1, ..., size_t dim_n$	

The method used to pass array arguments to FORTRAN 77 functions in the absence of an explicit external function call is similar to the one defined above for C: first the address of the array, then the dimension sizes as integers. See the table below. The type  $T$  is allowed to be any of the simple types which can be passed to FORTRAN 77 as defined in section 6.2.1 and it is mapped to the type  $T'$  as defined in that section.

Modelica	FORTRAN 77
	Input and Output
$T[dim_1]$	$T', \text{ INTEGER } dim_1$
$T[dim_1, dim_2]$	$T', \text{ INTEGER } dim_1, \text{ INTEGER } dim_2$
$T[dim_1, \dots, dim_n]$	$T', \text{ INTEGER } dim_1, \dots, \text{ INTEGER } dim_n$

[The following two examples illustrate the default mapping of array arguments to external C and FORTRAN 77 functions.

```
function foo
  input  Real      a[:,:,:];
  output Real      x;
  external;
end foo;
```

The corresponding C prototype is as follows:

```
double foo(double *, size_t, size_t, size_t);
```

If the external function is written in FORTRAN 77, i.e.:

```
function foo
  input  Real      a[:,:,:];
  output Real      x;
  external "FORTRAN 77";
end foo;
```

the default assumptions correspond to a FORTRAN 77 function defined as follows:

```
FUNCTION foo(a, d1, d2, d3)
  DOUBLE PRECISION(d1,d2,d3) a
  INTEGER              d1
  INTEGER              d2
  INTEGER              d3
  DOUBLE PRECISION      foo
  ...
END
```

]

When an explicit call to the external function is present, the array and the sizes of its dimensions must be passed explicitly.

[This example shows how to arrays can be passed explicitly to an external FORTRAN 77 function when the default assumptions are unsuitable.

```
function foo
  input  Real      x[:];
  input  Real      y[size(x,1),:];
  input  Integer   i;
  output Real     u1[size(y,1)];
  output Integer  u2[size(y,2)];
  external "FORTRAN 77" myfoo(x, y, size(x,1), size(y,2),
                               u1, i, u2);
end foo;
```

The corresponding FORTRAN 77 subroutine would be declared as follows:

```
SUBROUTINE myfoo(x, y, n, m, u1, i, u2)
  DOUBLE PRECISION(n)    x
  DOUBLE PRECISION(n,m)  y
```

```

  INTEGER          n
  INTEGER          m
  DOUBLE PRECISION(n)  u1
  INTEGER          i
  DOUBLE PRECISION(m)  u2
  ...
END

```

This example shows how to pass an array in column major order to a C function.

```

function fie
  input  Real[:,:] a;
  output Real b;
  external;
  annotation(arrayLayout = "columnMajor");
end fie;

```

This corresponds to the following C-prototype:

```

double fie(double *, size_t, size_t);
]

```

### 6.2.3 Records

Mapping of record types is only supported for C. A Modelica record class that contains simple types, other record elements, or arrays with fixed dimensions thereof, is mapped as follows:

- The record class is represented by a struct in C.
- Each element of the Modelica record is mapped to its corresponding C representation. The elements of the Modelica record class are declared in the same order in the C struct.
- Arrays are mapped to the corresponding C array, taking the default array layout or any explicit `arrayLayout`-directive into consideration.
- Records are passed by reference (i.e. a pointer to the record is being passed).

For example,

```

record R
  Real x;
  Integer y[10];      is mapped to
  Real z;
end R;

```

```

struct R {
  double x;
  int y[10];
  double z;
};

```

## 6.3 Return type mapping

If there is a single output parameter and no explicit call of the external function, or if there is an explicit external call in the form of an equation, in which case the LHS must be one of the output parameters, the external routine is assumed to be a value-returning function. Mapping of the return type of functions is performed as indicated in the table below. Storage for arrays as return values is allocated by the calling routine, so the dimensions of the returned array are fixed at call time. Otherwise the external function is assumed not to return anything; i.e., it is really a procedure or, in C, a `void`-function. [In this case, argument type mapping according to section 6.2 is performed in the absence of any explicit external function call.]

Return types are by default mapped as follows for C and FORTRAN 77:

Modelica	C	FORTRAN 77
Real	double	DOUBLE PRECISION
Integer	int	INTEGER

Boolean	int	LOGICAL
$T[ dim_1, \dots, dim_n ]$	$T' *$	$T'$
Record	See section 6.2.3.	Not allowed.

The element type  $T$  of an array can be any simple type as defined in section 6.2.1 or, for C, a record type as defined in section 6.2.3. The element type  $T$  is mapped to the type  $T'$  as defined in these sections.

## 6.4 Aliasing

Any potential aliasing in the external function is the responsibility of the tool and not the user. An external function is not allowed to internally change the inputs (even if they are restored before the end of the function).

*[Example:*

```
function foo
  input Real x;
  input Real y;
  output Real z:=x;
  external "FORTRAN 77" myfoo(x,y,z);
end foo;
```

*The following Modelica function:*

```
function f
  input Real a;
  output Real b;
algorithm
  b:=foo(a,a);
  b:=foo(b,2*b);
end f;
```

*can on most systems be transformed into the following C function*

```
double f(double a) {
  extern void myfoo_(double*,double*,double*);
  double b,temp1,temp2;
  myfoo_(&a,&a,&b);
  temp1=2*b;
  temp2=b;
  myfoo_(&b,&temp1,&temp2);
  return temp2;
}
```

*The reason for not allowing the external function to change the inputs is to ensure that inputs can be stored in static memory and to avoid superfluous copying (especially of matrices). If the routine does not satisfy the requirements the interface must copy the input argument to a temporary. This is rare but occurs e.g. in dormlq in some Lapack implementations. In those special cases the writer of the external interface have to copy the input to a temporary. If the first input was changed internally in myfoo the designer of the interface would have to change the interface function “foo” to:*

```
function foo
  input Real x;
  protected Real xtemp:=x; // Temporary used because myfoo changes its input
  public input Real y;
  output Real z;
  external "FORTRAN 77" myfoo(xtemp,y,z);
end foo;
```

*Note that we discuss input arguments for Fortran-routines even though Fortran 77 does not formally have input arguments and forbid aliasing between any pair of arguments to a function (section 15.9.3.6 of X3J3/90.4). For the few (if any) Fortran 77 compilers that strictly follow the standard and are unable to handle aliasing between input variables the tool must transform the first call of foo into*

```
temp1=a; /* Temporary to avoid aliasing */
myfoo_(&a,&temp1,&b);
```

*The use of the function foo in Modelica is uninfluenced by these considerations.]*

## 6.5 Examples

### 6.5.1 Input parameters, function value

*[Here all parameters to the external function are input parameters. One function value is returned. If the external language is not specified, the default is "C", as below.*

```
function foo
  input Real x;
  input Integer y;
  output Real w;
  external;
end foo;
```

*This corresponds to the following C-prototype:*

```
double foo(double, int);
```

*Example call in Modelica:*

```
z = foo(2.4, 3);
```

*Translated call in C:*

```
z = foo(2.4, 3);
```

### 6.5.2 Arbitrary placement of output parameters, no external function value

*In the following example, the external function call is given explicitly which allows passing the arguments in a different order than in the Modelica version.*

```
function foo
  input Real x;
  input Integer y;
  output Real u1;
  output Integer u2;
  external "C" myfoo(x, u1, y, u2);
end foo;
```

*This corresponds to the following C-prototype:*

```
void myfoo(double, double *, int, int *);
```

*Example call in Modelica:*

```
(z1,i2) = foo(2.4, 3);
```

*Translated call in C:*

```
myfoo(2.4, &z1, 3, &i2);
```

### 6.5.3 External function with both function value and output variable

*The following external function returns two results: one function value and one output parameter value. Both are mapped to Modelica output parameters.*

```
function foo
  input Real x;
  input Integer y;
  output Real funcvalue;
  output Integer out1;
  external "C" funcvalue = myfoo(x, y, out1);
end foo;
```

*This corresponds to the following C-prototype:*

```
double myfoo(double, int, int *);
```

*Example call in Modelica:*

```
(z1,i2) = foo(2.4, 3);
```

*Translated call in C:*

```
z1 = myfoo(2.4, 3, &i2);
```

```
l
```

---

## 7 Modelica standard library

---

The pre-defined, free "**package** Modelica" is shipped together with a Modelica translator. It is an extensive standard library of pre-defined components in several domains. Furthermore, it contains a standard set of **type** and **interface** definitions in order to influence the trivial decisions of model design process. If, as far as possible, standard quantity types and connectors are relied on in modeling work, model compatibility and thereby reuse is enhanced. Achieving model compatibility, without having to resort to explicit coordination of modeling activities, is essential to the formation of globally accessible libraries. Naturally, a modeller is not required to use the standard library and may also add any number of local base definitions.

The library will be amended and revised as part of the ordinary language revision process. It is expected that informal standard base classes will develop in various domains and that these gradually will be incorporated into the Modelica standard library.

The type definitions in the library are based on ISO 31-1992. Several ISO quantities have long names that tend to become awkward in practical modeling work. For this reason, shorter alias-names are also provided if necessary. Using, e.g., "ElectricPotential" repeatedly in a model becomes cumbersome and therefore "Voltage" is also supplied as an alternative.

The standard library is not limited to pure SI units. Whenever common engineering practice uses a different set of (possibly inconsistent) units, corresponding quantities will be allowed in the standard library, for example English units. It is also frequently common to write models with respect to scaled SI units in order to improve the condition of the model equations or to keep the actual values around one for easier reading and writing of numbers.

The connectors and partial models have predefined graphical attributes in order that the basic visual appearance is the same in all Modelica based systems.

The complete Modelica package can be downloaded. Below, the introductory documentation of this library is given together with links to the subpackages. Note, that the Modelica package is still under development.

```
package Modelica
  package Info
    /* The Modelica package is a standardized, pre-defined and free
       package, that is shipped together with a Modelica translator. The
       package provides constants, types, connectors, partial models and
       model components in various disciplines.
```

In the Modelica package the following conventions are used:

- Class and instance names are written in upper and lower case letters, e.g., "ElectricCurrent". An underscore is only used at the end of a name to characterize a lower or upper index, e.g., body\_low\_up.
- Type names start always with an upper case letter.  
Instance names start always with a lower case letter with only a few exceptions, such as "T" for a temperature instance.
- A package XXX has its interface definitions in subpackage XXX.Interface, e.g., Electric.Interface.
- Preferred instance names for connectors:

p,n: positive and negative side of a partial model.  
 a,b: side "a" and side "b" of a partial model  
 (= connectors are completely equivalent).

The following subpackages are available:

### GENERAL PACKAGES

<u>Constant</u>	Mathematical and physical constants
<u>Math</u>	Mathematical functions
<u>SIunit</u>	SI-unit type definitions

### FORMALISM PACKAGES

<u>BlockDiagram</u>	Input/output blocks
<u>BondGraph</u>	Bond graph components
<u>FiniteStateMachine</u>	Finite state machine
<u>PetriNet</u>	One-token petri-nets.

### GENERAL DOMAINS

<u>Electric</u>	Electric and electronic components
<u>Mechanics</u>	1D and 3D mechanical components
<u>ThermoFluid</u>	1D thermo-fluid components

### DOMAIN PACKAGES

<u>Aircraft</u>	Aircraft components
<u>Building</u>	Energy balance of building components
<u>DriveTrain</u>	Planetary gearboxes, clutches
<u>ElectricPower</u>	Generators, motors, electric line
<u>Hydraulics</u>	Hydraulic components

\*/

```
end Info;
end Modelica;
```